Uncertainty and heavy tails in EU stock markets before and during the financial crisis

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Abstract
In this paper we are using daily data for main stock market indexes of EU countries in order to study the uncertainty behaviour of these stock markets before and during financial crisis.
Our result shows a significant difference in the uncertainty pattern of EU stock markets before and during financial crisis. Also, there is a large variability among EU stock markets in uncertainty patterns and this fact could be explained trough inequalities in stock market and economic development.

Keywords: stable distributions, financial crisis, uncertainty, entropy
JEL Classification Codes: G170

1. Introduction

Financial crisis had a severe impact on stock markets across European Union, following the pattern from US stock market. This impact could be revealed using a key feature of financial crisis, a higher level of uncertainty.

The level of uncertainty of a stock market could be analyzed using two complementary approaches: information theory and statistical modelling. From information theory point of view, a robust measure of uncertainty is entropy; in classical form, the Shannon entropy is positively correlated to the level of uncertainty.

Also, using the statistical approach, the probability in the tails of the returns' distribution is an estimate of uncertainty level of a stock market.

As a measure of stock market uncertainty we are using entropy of distribution function of returns, while the tail behaviour of returns' distribution is captured using alpha-stable distributions.

Knowing the probability distribution of returns is essential for any statistical inference made about the stock market. In general, it is considered that major distribution characterizing the evolution of returns is the normal distribution (Gaussian) or its derivatives (e.g. log-normal distribution).

More recent papers (Rachev et al., 2000 and 2010) show that stable distributions are a much better approach than classical distributions in financial modeling. The fact that the observed distribution of returns is heavy-tailed can not be explained by a normal distribution.

The relationship between stable distributions and financial crisis has been addressed by Barunik, Vacha and Vosvrda (2010). In this study, they are estimating parameters of stable distributions or US and Central Europe stock markets, using daily and intraday data. Analyzing the distribution of returns for 2005-2009, and separately for the periods 2005-2007 (before the financial crisis) and 2007-2009 (the crisis), the authors conclude that there is a significant difference between the probability distribution of returns before and during the financial crisis.

Thus, the pre-financial crisis presents no large deviation from normal distribution, while the crisis is characterized by a significant deviation from normality.

In this paper we are using daily data for main stock market indexes of EU-27 countries in order to study the uncertainty behaviour of these stock markets before and during financial crisis.

The study is structured as follows: first section contains a theoretical presentation of measures of uncertainty used, in the second section the main results are presented and the last section is for conclusions.

2. Measures of uncertainty

2. 1. Stable distributions

Stable distributions are a class of distributions which have the property of being invariant under linear combinations; Gaussian distribution is a special case of stable distributions.

The difficulty that occurs for stable distributions is that in most cases is not known an explicit form of probability density function, but only the expression of characteristic function. Thus, a random variable X follows a stable distribution with parameters $(\alpha, \beta, \gamma, \delta)$ (Nolan,2011) if exists $\gamma > 0, \delta \in \Re$ such as X and $\gamma Z + \delta$ have the same distribution, where Z is a random variable with characteristic function

$$\phi(t) = \mathbf{E}[e^{itZ}] = \begin{cases} \exp(-|t|^{\alpha}[1 - i\beta \tan(\frac{\pi\alpha}{2}) sign(t)]), \alpha \neq 1 \\ \exp(-|t|[1 + i\beta t \frac{2}{\pi} sign(t)(\ln(|t|)]), \alpha = 1 \end{cases}.$$

In the above notations $\alpha \in (0.2]$ is the stability index, controlling for probability in the tails(for Gaussian distribution $\alpha = 2$), $\beta \in [-1,1]$ is the skewness parameter, $\gamma \in (0, \infty)$ is the scale parameter and $\delta \in \mathbf{R}$ is the location parameter.

A random variable X follows a stable distribution $S(\alpha, \beta, \gamma, \delta; 0)$ if his characteristic function has the form

$$\phi(t) = \mathbf{E}[e^{itX}] = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 + i\beta \tan(\frac{\pi\alpha}{2}) sign(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t), \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t \frac{2}{\pi} sign(t)(\ln(|\gamma t|)] + i\delta t), \alpha = 1 \end{cases}. \quad (1)$$

A random variable X follows a stable distribution $S(\alpha, \beta, \gamma, \delta; 1)$ if his characteristic function has the form

$$\phi(t) = \mathbf{E}[e^{itX}] = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 - i\beta \tan(\frac{\pi\alpha}{2}) sign(t)] + i\delta t), \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t \frac{2}{\pi} sign(t)(\ln(|t|)] + i\delta t), \alpha = 1 \end{cases}. \quad (2)$$

Parametrisation $S(\alpha, \beta, \gamma, \delta; 1)$ has the asdvatage thas is more suitable for algebric manipulations, altough his characteristic function is not continuous for all parameters.

Parametrisation $S(\alpha, \beta, \gamma, \delta; 0)$ is suitable for numerical simulations and statistical inference, altough the expression of characteristic function is more difficult to utilise in algebric calculus.

Nolan(2011) shows that the two parametrisations are equivalent; if $X \sim S(\alpha, \beta, \gamma, \delta_1; 1)$ and $X \sim S(\alpha, \beta, \gamma, \delta_0; 0)$, then

$$\delta_0 = \begin{cases} \delta_1 + \beta\gamma \tan\frac{\pi\alpha}{2}, \alpha \neq 1 \\ \delta_1 + \beta\frac{2}{\pi}\gamma \ln\gamma, \alpha = 1 \end{cases}. \quad (3)$$

The behavior of stable distributions is driven by the values of stability index α : small values are associated to higher probabilities in the tails of the distribution.

2.2. Information entropy

Information entropy is the most widely used measure of uncertainty, applications covering a wide range, from physics to economics and biology. The concept of entropy originates from physics in the 19th century; the second law of thermodynamics stating that the entropy of a system cannot decrease other way than by increasing the entropy of another system. As a consequence, the entropy of a system in isolation can only increase or remain constant over time. If the stock market is regarded as a system, then it is not an isolated system: there is a constant transfer of information between the stock market and the real economy. Thus, when information arrives from (leaves to) the real economy, then we can expect to see an increase (decrease) in the entropy of the stock market, corresponding to situations of increased (decreased) randomness.

Most often, entropy is used in one of the two main approaches, either as Shannon Entropy – in the discrete case – or as Differential Entropy – in the continuous time case. Shannon Entropy quantifies the expected value of information contained in a realization of a discrete random variable. Also, is a measure of uncertainty, or unpredictability: for a uniform discrete distribution, when all the values of the distribution have the same probability, Shannon Entropy reaches his maximum. Minimum value of Shannon Entropy[1] corresponds to perfect predictability, while higher values of Shannon Entropy correspond to lower degrees of predictability.

Differential Entropy is an extension of Shannon Entropy to the continuous case, but is not a good measure of uncertainty; can take negative values and in addition is not invariant to some linear transformations.

Dioniso et al. (2006) provide a review of the theoretical and empirical work about the entropy and the variance as measures of uncertainty. Several conclusions could be drawn from this review: first of all, the entropy is a more general measure of uncertainty than he variance or the standard deviation (Philippatos and Wilson, 1972), since the entropy depends on more characteristics of a distribution as compared to the variance and may be related to the higher moments of a distribution (Ebrahimi et al., 1999). Secondly, both the entropy and the variance reflect the degree of concentration for a particular distribution, but their metric is different; while the variance measure the concentration around the mean, the entropy measures the diffuseness of the density irrespective of the location parameter (Ebrahimi, Maasoumi and Soofi, 1999).

In this paper we use a recently developed concept, the entropy of a function (Lorentz, 2009) in order to estimate the entropy of a distribution function, under very general assumptions and in a non-parametric context.

Basically, our methodology involves the following steps to estimate the entropy of a distribution function(Lazar et al.(2011)), for a sample $X_0,....,X_{n-1}$ of i.i.d. observations drawn form the distribution *F*:

[1] The minimum value of Shannon Entropy is 0.

Step 1. Estimate the distribution function using a Kernel Estimator or Empirical Distribution Estimator, obtaining values $\hat{F}_n(X_i)$ for $i = 0,..,n-1$;

Step 2. Sample from the distribution function, using the sampled function $S_n(\hat{F}_n)(i) = \hat{F}_n(X_i)$ for $i = 0,..,n-1$;

Step 3. Define a quantum $q > 0$; then $Q_q S_n(\hat{F}_n)(j) = (i + 1/2)q$,

if $\hat{F}_n(X_j) \in [iq, (i+1)q)$;

Step 4. Compute the probabilities

$$p_n(i) = \frac{c_n(i)}{\sum_j c_n(j)} = \frac{c_n(i)}{n} = \frac{card\{\hat{F}_n(X_j) \in [iq, (i+1)q)\}}{n};$$

Step 5. The estimator of entropy of distribution function is then

$$H_q(\hat{F}_n) = -\sum_i p_n(i) \log_2 p_n(i). \quad (4)$$

In order to insure comparability among various distributions, one can define a normalized entropy, as a ratio between the entropy and the entropy of uniform distribution:

$$NH_q(\hat{F}_n) = -\sum_i p_n(i) \log_2 p_n(i) / \log_2 n \in [0,1]. \quad (5)$$

In the following we will refer to entropy as the normalized entropy, taking values between 0 and 1.

Low values of entropy are associated with heavy-tailed distributions, while high values of entropy correspond to Gaussian distribution; in other words, as the tails probability is higher, the expected value of entropy is lower.

3. Data and empirical results

In order to asses the impact of financial crisis on uncertainty behavior of stock market indexes for EU countries, we use a sample of daily observations for 26 european countries.

Starting from observed price p_t, we compute the logreturns as $r_t = \log p_t - \log p_{t-1}$ and using the methodology described above, we estimate entropy of distribution function of returns and also the stability index α for stable distribution[2].

Table 1. Stock market indexes

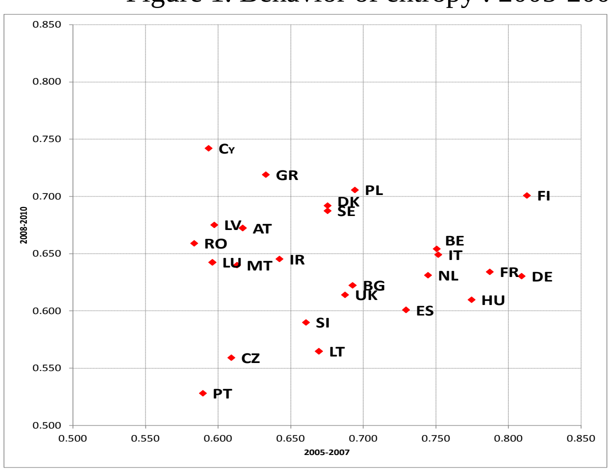
Country	Country code	Index	2005-2007 Entropy	2008-2010 Entropy	2005-2007 α	2008-2010 α
Malta	MT	MSE	0.613	0.640	1.283	1.373
Bulgaria	BG	SOFIX	0.693	0.622	1.414	1.371
Slovenia	SI	SBITOP	0.661	0.590	1.482	1.501
Cyprus	CY	CYSMMAPA	0.593	0.742	1.578	1.809
Lithuania	LT	VILSE	0.669	0.565	1.590	1.443
Portugal	PT	PSI20	0.589	0.528	1.629	1.592
Ireland	IR	ISEQ	0.642	0.645	1.641	1.682
Latvia	LV	RIGSE	0.597	0.675	1.663	1.695

[2] We use STABLE.EXE, available on http://academic2.american.edu/~jpnolan/stable/stable.html .

Sweden	SE	OMX Stockholm	0.675	0.687	1.668	1.655
Denmark	DK	OMX Copenhagen	0.675	0.692	1.668	1.657
Romania	RO	BET	0.584	0.659	1.680	1.637
UK	UK	FTSE100	0.687	0.614	1.682	1.598
Belgium	BE	BEL20	0.751	0.654	1.706	1.714
Czech Republic	CZ	PX50	0.609	0.559	1.718	1.572
Austria	AT	ATX20	0.617	0.672	1.729	1.672
Greece	GR	ASE	0.633	0.719	1.741	1.808
Luxemburg	LU	LUXX	0.596	0.642	1.770	1.738
Netherlands	NL	AEX	0.744	0.631	1.773	1.566
Spain	ES	IBEX	0.729	0.601	1.776	1.684
Finland	FI	OMXH15	0.813	0.701	1.805	1.724
Poland	PL	WIG	0.694	0.706	1.854	1.667
France	FR	CAC40	0.787	0.634	1.857	1.648
Italy	IT	FTSEMIB	0.752	0.649	1.858	1.671
Hungary	HU	BUX	0.775	0.610	1.871	1.699
Germany	DE	DAX	0.809	0.630	1.881	1.620

The results obtained for the two subsamples analyzed shows significant differences between EU countries from the point of view of stock market uncertainty.

Figure 1. Behavior of entropy : 2005-2007 vs 2008-2010



Thus, for 2005-2007 period, before the financial crisis, one can distinguish three clusters of countries, based on entropy behavior:

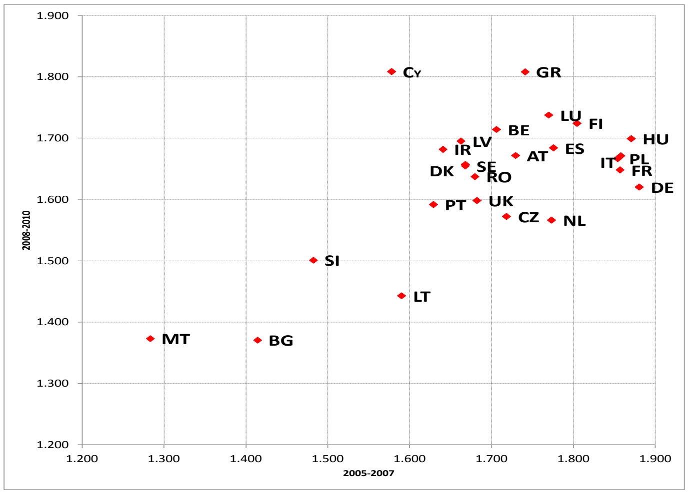
- first cluster – countries with low uncertainty(high entropy) is formed by the following countries: Finland, Germany, France, Hungary, Netherlands, Italy, Belgium, Spain;
- second cluster – countries with moderate uncertainty is formed by the following countries: Lithuania, UK, Bulgaria, Poland, Denmark, Sweden, Slovenia;
- third cluster – countries with high uncertainty is formed by the following countries: Portugal, Greece, Czech Republic, Malta, Luxembourg, Romania, Cyprus, Latvia, Ireland.

The situation is slighly different in period 2008-2008(financial crisis): for the entire sample of countries, entropy is lower than in period 2005-2007, indicating an increased likelihood of extreme events on stock market and a higher degree of uncertainty.

Thus, for 2008-20010 period, during the financial crisis, one can distinguish three clusters of countries, based on entropy behavior:

- first cluster – countries with low uncertainty(high entropy) is formed by the following countries: Greece, Cyprus, Poland, Finland;
- second cluster – countries with moderate uncertainty is formed by the following countries: Germany, France, Hungary, Netherlands, Italy, Belgium, Spain, UK, Bulgaria, Denmark, Sweden, Greece, Malta, Luxembourg, Romania, Cyprus, Ireland;
- third cluster – countries with high uncertainty is formed by the following countries: Portugal, Czech Republic, Lithuania, Slovenia.

Figure 2. Behavior of α (stability index) : 2005-2007 vs 2008-2010



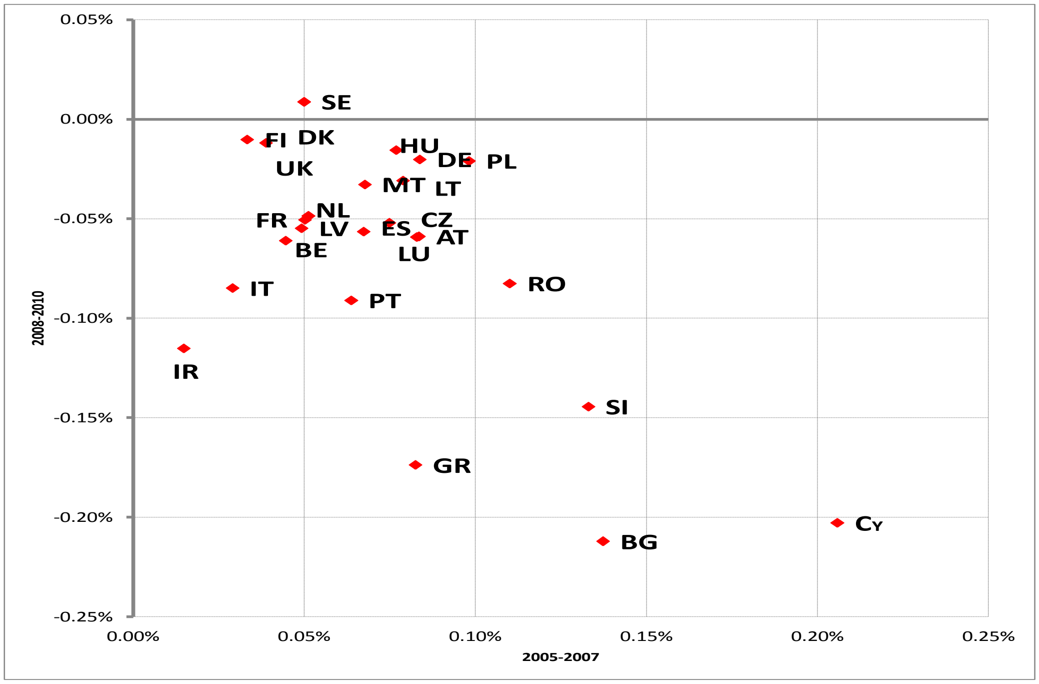
Estimation of stability index α of the stable distribution shows a different clustering behavior of EU stock markets: for most of the countries, there is a departure from normality induced by the financial crisis. Two extreme clusters could be identified based on this criterion:

- countries with extreme departure from normality: Malta, Bulgaria, Slovenia, Lithuania;
- countries for which stability index α shows a behavior close to Gaussian distribution: Greece and Cyprus.

Table 2. Return and volatility

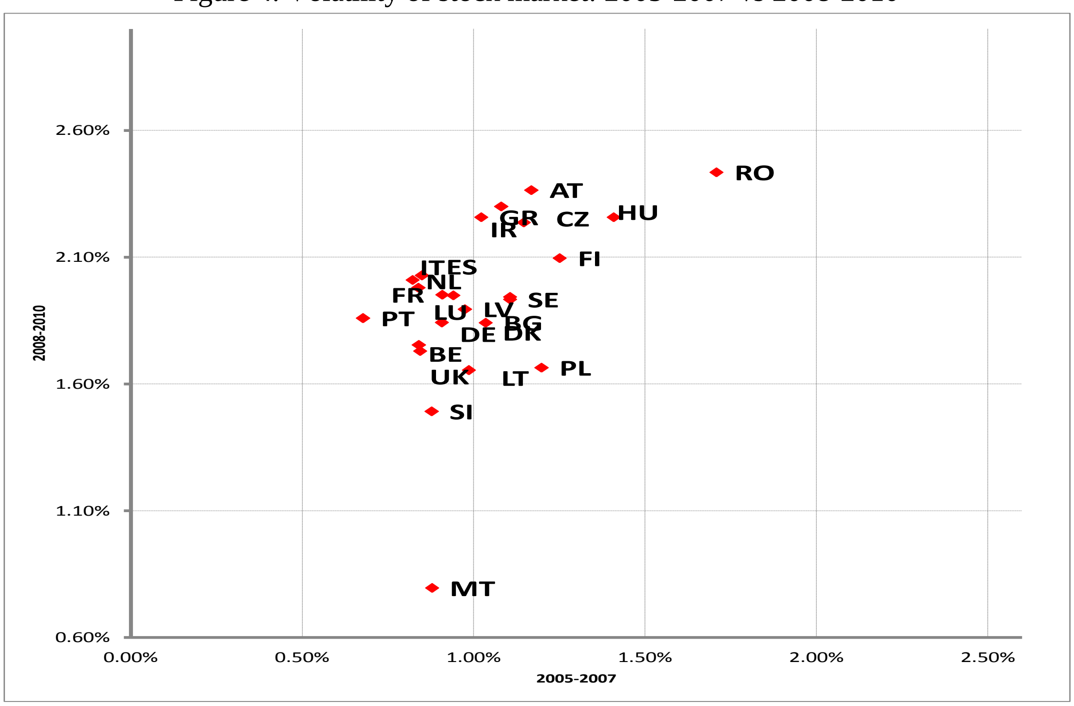
Country	Country code	Index	2005-2007 Return	2008-2010 Return	2005-2007 Volatility	2008-2010 Volatility
Malta	MT	MSE	0.068%	-0.033%	0.879%	0.795%
Bulgaria	BG	SOFIX	0.137%	-0.212%	1.035%	1.841%
Slovenia	SI	SBITOP	0.133%	-0.144%	0.878%	1.492%
Cyprus	CY	CYSMMAPA	0.206%	-0.203%	1.523%	3.021%
Lithuania	LT	VILSE	0.079%	-0.031%	0.986%	1.654%
Portugal	PT	PSI20	0.064%	-0.091%	0.678%	1.859%
Ireland	IR	ISEQ	0.015%	-0.115%	1.081%	2.300%
Latvia	LV	RIGSE	0.049%	-0.055%	0.975%	1.895%
Sweden	SE	OMX Stockholm	0.050%	0.009%	1.106%	1.933%
Denmark	DK	OMX Copenhagen	0.050%	0.009%	1.106%	1.943%
Romania	RO	BET	0.110%	-0.083%	1.709%	2.434%
UK	UK	FTSE100	0.039%	-0.012%	0.844%	1.730%
Belgium	BE	BEL20	0.045%	-0.061%	0.841%	1.754%
Czech Republic	CZ	PX50	0.075%	-0.052%	1.146%	2.237%
Austria	AT	ATX20	0.083%	-0.059%	1.169%	2.365%
Greece	GR	ASE	0.083%	-0.174%	1.022%	2.257%
Luxemburg	LU	LUXX	0.083%	-0.059%	0.941%	1.949%
Netherlands	NL	AEX	0.051%	-0.049%	0.840%	1.979%
Spain	ES	IBEX	0.067%	-0.057%	0.850%	2.027%
Finland	FI	OMXH15	0.033%	-0.010%	1.252%	2.096%
Poland	PL	WIG	0.098%	-0.021%	1.199%	1.664%
France	FR	CAC40	0.050%	-0.051%	0.909%	1.952%
Italy	IT	FTSEMIB	0.029%	-0.085%	0.822%	2.010%
Hungary	HU	BUX	0.077%	-0.016%	1.408%	2.258%
Germany	DE	DAX	0.084%	-0.020%	0.908%	1.842%

Figure 3. Performance of stock market: 2005-2007 vs 2008-2010



Analysis of european stock markets could be performed also in terms of performance(returns) and volatility; what financial crisis brings from point of view of stock market performance is a significant drop in average daily returns and also a significant increase in volatility.

Figure 4. Volatility of stock market: 2005-2007 vs 2008-2010



From the point of view of average daily returns before and during the financial crisis, most of european stock markets have a similar behavior, with few notable exceptions: Greece, Cyprus, Bulgaria and Slovenia. For these countries the financial crisis had a severe impact on stock market performance; for example, Cyprus had a drop in average daily return from 0.2% to -0.2%, while Greece average daily return dropped from 0.08% to -1.7%.
Also in terms of volatility, both before and during financial crisis, we can observe a cluster of countries with the highest volatilities: Greece, Austria, Romania, Ireland, Hungary, Czech Republic, Finland, Hungary.

As the above results shows, the impact of financial crisis on european stock markets was not uniform across countries; there are significant differences in terms of performance, volatility, uncertainty and behavior towards normality.
Yet, in order to realize a bettter classification of european stock markets, according to those four criteria, we aplly a Principal Component Analysis(PCA), for the two sub-samples, 2005-2007 and 2008-2010.

Table 3. Total variance explained by PCA(2005-2007)

Component		Initial Eigenvalues			Rotation Sums of Squared Loadings		
		Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
	1	1.936	48.391	48.391	1.658	41.445	41.445
	2	1.170	29.259	77.649	1.448	36.204	77.649

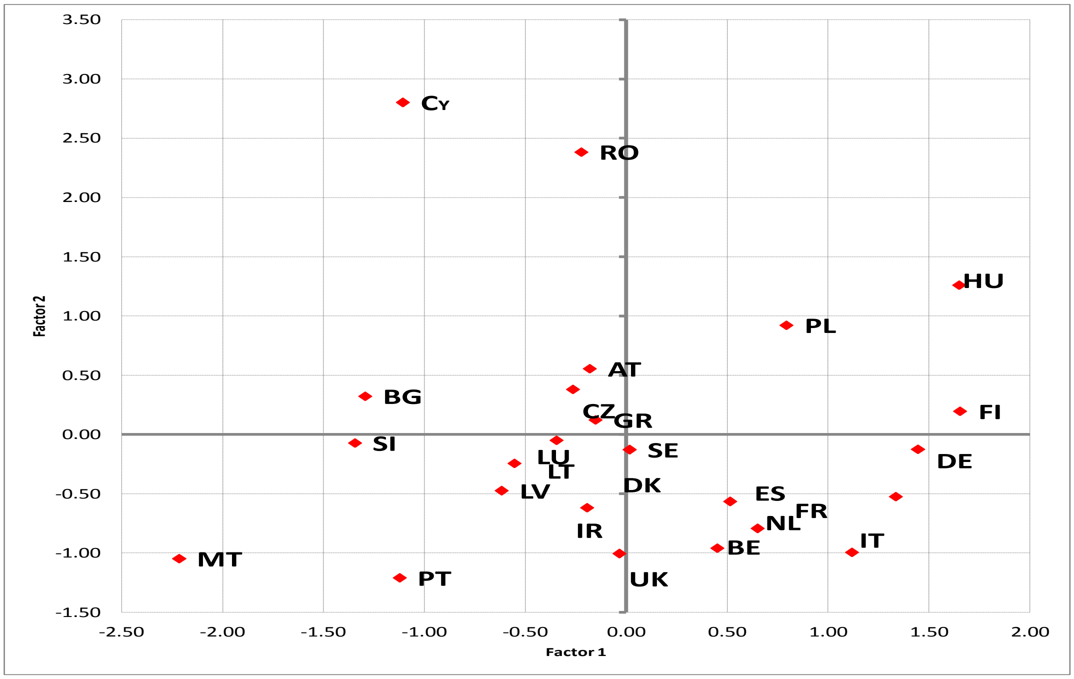
For the first sub-sample, before financial crisis, we obtain two principal components explaining togheter 77% of total variance.

Table 4. Rotated component matrix(2005-2007)

Variable	Component	
	1	2
Entropy	.799	-.240
α	.911	.057
Return	-.426	.729
Volatility	.090	.925

Our data could be represented in a 2-dimensional space, for which the first component is related to uncertainty(entropy and stability index), while the second component captures stock market return and stock market volatility.

Figure 4. Principal Component Analysis 2005-2007



Plotting the countries using the two component extracted with PCA, we can observe a compact group of countries near the origin of axes, having almost the same behavior in terms of variables analyzed, but also some outliers: Cyprus and Romania, with high values for the second component, Malta with low value for the first component.

Table 5. Total variance explained by PCA(2008-2010)

Component		Initial Eigenvalues			Rotation Sums of Squared Loadings		
		Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
	1	2.128	53.210	53.210	2.009	50.216	50.216
	2	1.053	26.327	79.537	1.173	29.321	79.537

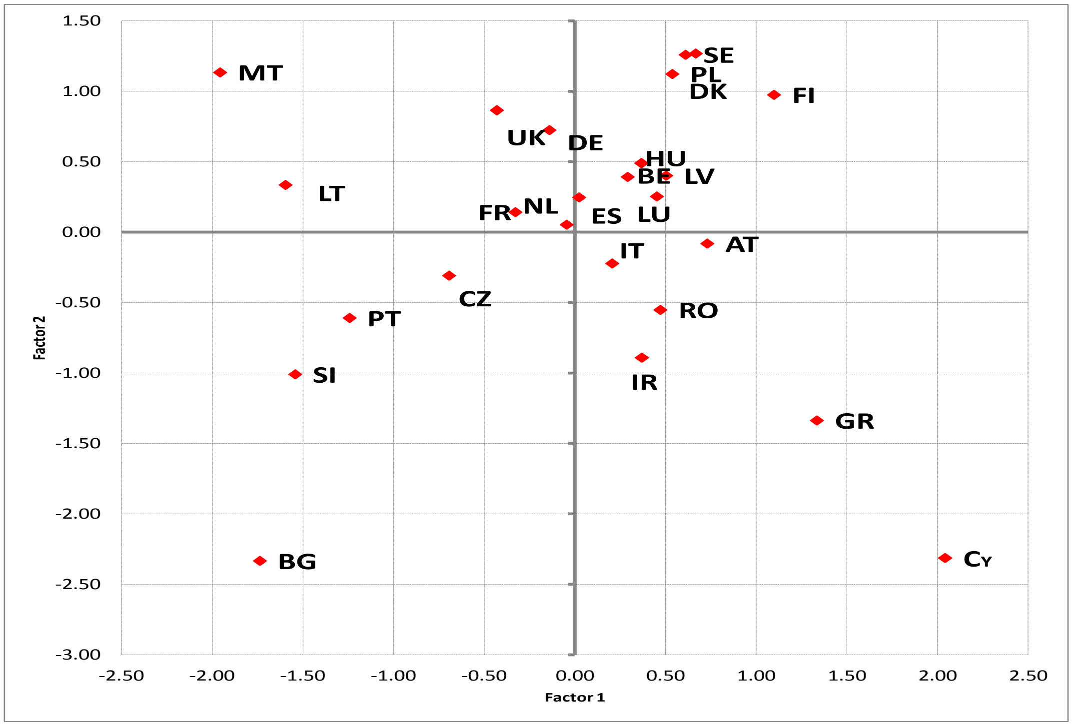
For the second sub-sample, during financial crisis, we obtain two principal components explaining togheter almost 80% of total variance.

Table 6. Rotated component matrix(2008-2010)

Variable	Component	
	1	2
Entropy	.786	.024
α	.930	-.004
Return	.001	.967
Volatility	.726	-.488

Our data could be represented in a 2-dimensional space, for which the first component is related to uncertainty(entropy, stability index and volatility), while the second component captures stock market return.

Figure 5. Principal Component Analysis 2008-2010



The clusterization of countries using components extracted for 2008-2010 reveals a quite different pattern: while most of the countries tend to be homogeneous, we can find several countries with different behavior: Greece, Cyprus, Bulgaria and Malta.

Also, there is a large group of countries around origin of axes, with neutral behavior and another group(Sweden, Finland, Denmark, Poland) with high values for both components.

Conclusions

Using daily data for main stock market indexes of EU-27 countries, we have studied the behaviour of these stock markets before and during the financial crisis.

The analysis was conducted on two directions, looking for significant differences between properties of return distribution and also looking for homogenous groups of countries based on stock market indicators.

From a distributional point of view, most of the countries exhibit large departure from normality during financial crisis, values of stability index α being significantly lower than 2(the case of Gaussian distribution).

The same conclusion was revealed using entropy of distribution function of returns as an estimator of stock market uncertainty. For majority of countries form our sample, there is a clear movement towards high uncertainty levels during financial crisis.

Principal Component Analysis applied to there sample and to the four stock market indicators (entropy, stability index, return and volatility) reveal a complete different map of European stock markets before and during financial crisis. Thus, before the financial crisis, the level of homogeneity is higher, and there are just a few outliers based on the components extracted using Principal Component Analysis. During the financial crisis the heterogeneity among European stock markets is increasing and there are local clusters of countries with similar behaviour.

Further research need to be conducted in order to explain this large variability among European stock markets in terms of uncertainty patterns, perhaps trough existing inequalities in stock market and economic development.

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