PD BACKTEST EMPIRICAL STUDY ON CREDIT RETAIL PORTFOLIO[1]

Martin Švec
ČSOB, a. s.
Credit Risk Modelling Department
Radlická 333/150
15057 Prague 5
Czech Republic
e-mail: msvec@csob.cz

Abstract

This paper shows PD backtest for Basel II Internal Rating Based approach. Although question of appropriate PD backtest has not directly arisen from the current financial crisis, its importance is amplified by it. Backtesting PD as comparison of predicted PD and realized default rates is one of the three parts to test PD rating systems. The other two parts are measurements of discrimination (model power) and stability (population/variables changes) of the model. Since these two parts are usually involved in regular reviews of individual models, this document is devoted only to the backtest of PD. The paper covers description from the last Basel II directive (issued in June 2006), together with improvements suggested by other related papers. The description starts from Traffic Light Approach, continues with normal test and ends with possible improvements of these methods, like taking into account time dimension using correlation. The practical example of PD backtest for credit retail portfolio is shown. The results show that there are several pools which do not satisfy backtested methodology. These exceptions are explained and steps to prevent future similar situations done. Possible improvements have open space, since the Basel II directive lets certain degree of freedom for each bank and related national regulator.

Keywords: *PD backtest; Basel II; validation; traffic light approach; default probability*

JEL codes: *G21, G28*

1. Introduction

Banks are allowed from Basel II to compute capital requirements on their own, using IRB (Internal Rating Based) approach. For this, three basic models should be created by the bank. These models are *PD* model for estimating borrower's probability of default, *LGD* model for estimating loss in case the borrower will default and *EAD* model for estimating exposure with which borrower will default. These modeled values are used to compute capital requirement and each of the models should be tested in a three different steps. The first step is

[1] The opinions expressed in this paper are those of the author and do not necessarily reflect views shared by the Ceskoslovenska obchodni banka or its staff.

testing whether the performance of the model is sufficient, the second test is whether the portfolio on which the model is applied undergoes some changes in behaviour and the third is to test ex post whether the estimated value is the same as the realized one from the real data. This third test is called backtesting. This paper deals with backtesting of *PD* model, which means comparing estimated *PD* values used for capital requirements against ex post realized default rates. To compare these values, one can use different statistical or visualization techniques. There are techniques recommended by the Basel II directive (2006), however just for market risk. This document takes suggestions of the Basel II directive (in the rest of this paper the Basel II directive will be called "Directive") as a basis and shows possible way in which *PD* backtest can be realized on credit risk, using some improvements resulting from the suggestions of other related research papers. The results are shown on two examples of credit retail portfolio.

The next section describes in more details how *PD* backtest is suggested for market risk in the Directive, what is the main difference between market and credit risk from the *PD* backtest point of view, how related research works deal with *PD* backtest and describes data and terminology used within this paper. After that, TLA approach is given in Section 3, description of normal test in Section 4, incorporating correlation in Section 5, results in Section 6, discussion in Section 7 and conclusion in Section 8.

2. PD Backtest

2.1 PD Backtest in Basel II Directive

A framework to incorporate backtest of default probabilities (*PD*) into the internal models approach was written by Basel Committee (1996). This framework, which focused on market risk capital requirements, was addition to the Capital Accord of July 1988 and described three-zone approach. This approach consists of green, yellow and red zone and is sometimes called Traffic Lights Approach (TLA). It serves as quality indicator for generated *PD* estimates. The good predictions appear in green-zone, suspicious cases in yellow-zone and bad ones in red-zone. TLA as graphic visualization was compared to the normal test by Blochwitz et al. (2004), where influence of different *PD* values and differently correlated realized default rates on the number of cases in the wrong color zones was examined. The conclusion was that performance of both approaches is broadly equal, however, in general the TLA appears to be slightly more powerful while the normal test is slightly more robust with respect to correlation of default events and in time. These results of comparison were repeated in Bank of International Settlement's working paper (2005) and the three-zone approach seems to be there slightly preferred. However Committee also submitted that normal test can be in some cases better. TLA description was also repeated in the Directive. The Directive also admits that "*at present, different banks perform different types of backtesting comparisons and the standards of interpretation also differ somewhat across banks*".

2.2 From Market to Credit PD Backtest

The Directive deals with *PD* Backtest from the market risk point of view. Since market risk enables to compute value at risk at each working day, about 250 values are available per year. In credit risk, monthly approach is usually established and 12 values per year available to test one rating class (testing on the pool level). In case model or product consists of several pools, the backtest on the product level can be done and number of available observations in this case is given by 12 multiplied by the number of pools. However, there are some limitations connected with usage TLA on the product level, which will be explained inside this paper. The lack of data for credit backtest can be solved by using external data from rating agencies, simulations can be used to extend available data set or the longer period to backtest can be used, preferably over the whole economic cycle. We decided to rely on our internal monthly data, since even if there are just 12 values per year, these *PD* estimates and realized default rates are based on huge data sets (thousands of accounts). To emphasize backtest on last changes of portfolio, one year is used in this work.

2.3 PD Backtest Modifications in Related Works

Although the Directive suggests 3 colors, Basel Committee stated that this is the minimum satisfactory number, hence the number of color zones can be chosen higher. The reason why not to use two colors (green and red) is that such dichotomy has strict boundary between good and bad model and no space for anything between. There can be good models which could be tested as bad or bad models which could be tested as good. This is the reason, why at least one additional (middle) color should be used. This color tells us that the model can be suspicious, i.e. neither strictly good nor strictly bad and that deeper analysis should be done. Castermans et al. (2007) reported usage of five-zone approach and Blochwitz et al. (2005) suggested HSV color model which enables smooth transition between colors. These suggestions come from *PD* backtest for market risk with its 250 values per year, where there was space for increasing number of colors.

Basically, TLA is connected with the binomial distribution. Since this assumes independence of individual realizations, there were attempts to deal with incorporating of dependence effects using correlation, e.g. Tasche (2003) or Blochwitz et al. (2004). Tasche introduced method to determine correlation influence in the way of assigning rating class into the color zone. Although it was shown that increasing correlation influences the results, it was not so for datasets with 50 or less observation to be backtested, which is the case of credit risk. Castermans et al. (2007) reported the way how correlations could be incorporated into computing realized default rates. Blochwitz et al. (2004) dealt with taking into account correlation and default rate volatility. The influence of correlation is also examined by Blochwitz et al. (2005) or Ching et al. (2006). Ching et al. also overview methods to estimate correlations other than those prescribed by the Directive, since the Directive does not take into account different specifics of individual countries. Rauhmeier and Scheule described (2005) decomposition of mean square error between *PD* and realized default rates, which is closer to normal test than TLA approach.

2.4 Data and Terminology

Two products are used as examples of credit retail portfolio, they are denoted in this paper as product A and product B. Each product is divided into different number of pools, product A into 20 pools and product B into 16 pools. Each pool covers product's portfolio subset with similar risk profile. When the backtest is realized on the pool level, there are 12 values per year for each pool. When realized on the product level, *L* observations can be available over the tested year, where *L* is 12 times number of pools. Terminology used in the rest of this document is given in Table 1.

Table 1 Terminology

	Name	Description
PD	Probability of Default	A predicted Basel II default rate, different for each pool.
T_1	Time of *PD* estimate	Time at which *PD* is estimated.
T_2	Time of *PD* usage	Time at which estimated *PD* is applied to compute capital requirement (after 12 months period past *PD* estimation).
DF	Default Frequency	A natural number (not percentage) of defaults observed for T_2. If client is defaulted on any of his loans, all his other loans are also denoted as defaulted. If account was already defaulted in T_1, it is not used for *DF*.
AF	Account Frequency	A natural number (not percentage) of accounts observed at T_2. Accounts which were defaulted already at T_1 are not counted.
BDR	Backtest Default Rate	Realized default rate $\left(BDR = \frac{DF}{AF} \right)$. *BDR* is compared with *PD*.

Source: internal sources

The assignment of accounts to pools is defined at time T_1. If the account was not yet living at time T_1, it is assigned to pool at the moment of its first occurrence in the data. Note that this is one of the possible definitions of realized default rate and that before imposing eventual multiplication factor on the *PD* by business experts or national regulator (based on the results of *PD* backtest), there is necessity to understand the used definition of *BDR*.

3. Three-zone Traffic Lights Approach (*TLA*)

This section describes three-zone approach suggested in the Directive. The main point is whether estimated *PD* covers realized default rate (Backtest Default Rate - *BDR*) either for the whole product or for individual pools.

In case of individual pools, 12 observations are available for each pool, since the focus is on the latest available portfolio from the last year and since monthly data are available in the retail credit risk. The hypothesis is that the way in which *PD* was estimated for each pool is good. This means that *PD* is higher or equal to *BDR*. The case in which *PD* is less than *BDR* is called exception. Hence, for backtest on pool level, each pool can have from 0 to 12 exceptions. For backtest on product level, each product can have from 0 to *L* exceptions. Based on the number of exceptions, the whole product or individual pool can obtain three colors: green, yellow or red. The green one means that everything is ok, the yellow one means that further investigation and explanation should be done and the red one indicates that the model which produced *PD* estimate is probably bad.

Thinking in a way that there is a fix probability of exception for each observation (prescribed by the Directive) and under assumption of independence among individual observations, the process has binomial distribution. Binomial distribution with given number of observations N and given probability of exception *c* can be described by following equation:

$$p = F(e,c,N) = \frac{N!}{e!(N-e)!} c^{e} (1-c)^{(N-e)}, \qquad (1)$$

where $e = 0,1,2,\ldots N$.

Equation (1) computes probability (p) that exactly e exceptions will happen during N observations and under given probability (c) of having exception in a single observation. The probability c determines some “strictness” of the hypothesis that the model is good. Choosing higher c means that there is higher tendency to not reject the hypothesis about good model, since more exceptions are allowed than for lower probability c. The Basel Committee insists on 99% confidence interval, which means that in individual independent observation, probability *c* of having exception in a single observation is 1%. For more detail of the influence of *c* see Section 3.2 *TLA and Probability of Exception*.

3.1 TLA on Product and Pool Level

Using equation (1) for binomial distribution with 1% probability of exception, one can determine probability *p* with which certain number of exceptions can be realized and corresponding cumulative probability *cum_p*. These values for backtest on product level with hypothetical 250 observations can be seen in Table 2. For example, there is a probability of 6.66% that in 250 observations will be exactly 5 exceptions and that there is a probability of 95.88% that the number of exceptions will be less or equal to 5. The Directive defines boundaries between green, yellow and red zones. The green zone is for cumulative probability less than 95%, yellow for *cum_p* more or equal to 95% or less than 99.99% and the red one for cumulative probability equal or higher than 99.99%.

Although this might seem to be sufficient approach for backtesting on product level, it could be misleading: If the product has e.g. 16 pools and only one pool will be bad, this pool will generate 12 exceptions over the 12 months period and will push the whole product into the red zone (the red zone starts from 10 exceptions for product level). Rather than this approach, the backtest on the product level seems to be better when realized on the number of mean pool exception per month, which leads into 12 observations.

Table 2 Zones for TLA on product level

exceptions	*zone*	*p [%]*	*cum_p [%]*
0	g	8,11	8,11
1	g	20,47	28,58
2	g	25,74	54,32
3	g	21,49	75,81
4	g	13,41	89,22
5	y	6,66	95,88
6	y	2,75	98,63
7	y	0,97	99,60
8	y	0,30	99,89
9	y	0,08	99,97
10	r	0,02	99,99
11	r	0,00	100,00
12	r	0,00	100,00
….	r	…	…
….	r	…	…
250	r	0,00	100,00

Source: author's calculation

Table 3 Zones for TLA on pool level

exceptions	*zone*	*p [%]*	*cum_p [%]*
0	g	88,64	88,64
1	y	10,74	99,38
2	y	0,60	99,98
3	r	0,02	100,00
4	r	0,00	100,00
5	r	0,00	100,00
6	r	0,00	100,00
7	r	0,00	100,00
8	r	0,00	100,00
9	r	0,00	100,00
10	r	0,00	100,00
11	r	0,00	100,00
12	r	0,00	100,00

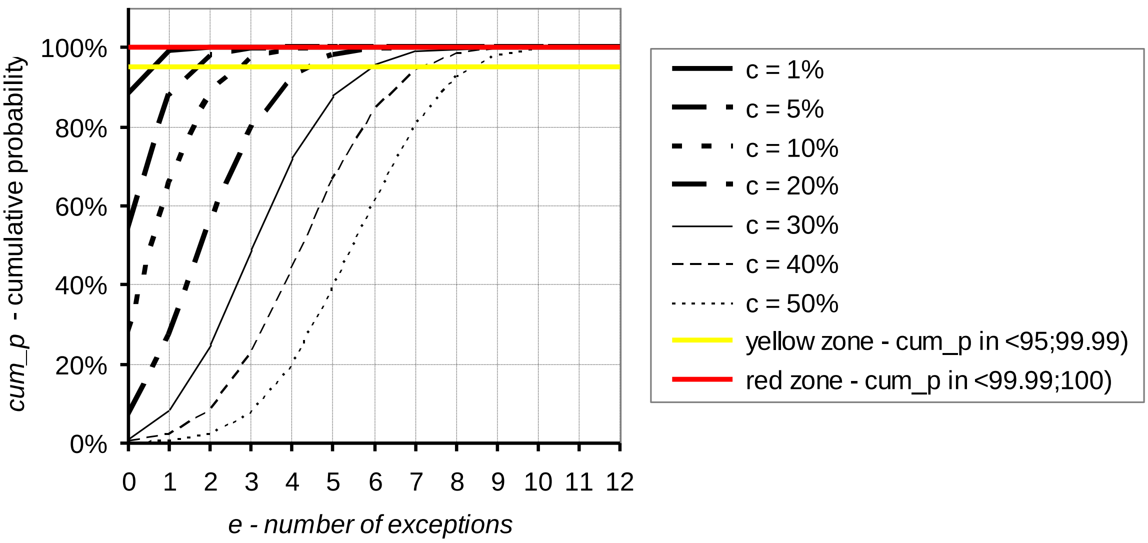
Source: author's calculation

For defining zones on 12 observations, see Table 3 above. Using equation (1), color zones are derived also for pool level. It can be seen that again all three zones are available. Based on the values of *cum_p*, the green zone allows no exception, the yellow zone 1 or 2 exceptions and the red zone covers more than 2 exceptions.

3.2 TLA and Probability of Exception

To see how definitions of the zones can be influenced by the probability of exception, see graph in Figure 1. To understand the graph, let us take for example yellow zone into account. The yellow zone is defined for cumulative probability higher or equal to 95% and lower than 99.99%. It can be seen from the above figure, that yellow zone crosses curve with *c*=1% already for 1 exception, which denotes such pool as suspicious. For *c*=5%, suspicious pool started at 2 exceptions and for *c*=50%, suspicious pools are those which have 9 exceptions from 12 observations. This example shows that the higher the probability *c*, the more exceptions are allowed to stay in the green zone (let us note again that the Directive chooses *c*=1%).

Figure 1 Influence caused by probability of exception (*c*) on the definition of color zones



Source: *author's calculation*

4. Normal Test

Since TLA approach can be rather considered as graphic visualization than a statistical test, normal test is performed to take into account statistic alternative. The principle, described in detail by Blochwitz et al. (2004), is that the hypothesis about good model ("All *BDR* are covered by *PD* so that *BDR* is less or equal to *PD*") can be reject at confidence level β if:

$$\frac{\sum_{t=1}^{N}\left(BDR_t - PD_t\right)}{\sqrt{N}\cdot\tau} > \beta\,, \tag{2}$$

where β is standard normal β-quantile (≈ 2.33 for 0.99 confidence level) and τ is the estimator of variance. Usual choice of τ is as

$$\tau_0^{\ 2} = \frac{1}{N-1}\sum_{t=1}^{N}\left(BDR_t - PD_t\right)^2\,, \tag{3}$$

which is however biased. For details, see Blochwitz et al. (2004). It is recommended to reduce bias using

$$\tau^2 = \frac{1}{N-1}\left(\sum_{t=1}^{N}\left(BDR_t - PD_t\right)^2 - \frac{1}{N}\left(\sum_{t=1}^{N}\left(BDR_t - PD_t\right)\right)^2\right). \tag{4}$$

For comparison, results for both biased and unbiased versions are given in Section 5 *Results*.

Equations (2) to (4) are common for both, pool and product backtest. The only difference is in the number of observations N, which can differ from pool to pool (smaller products can sometimes have missing pools in a few months, which is not the case of product examples showed in this paper) and from product to product (based on the number of pools for the product). There is no problem on the product level stated in Section 3, since even if only one pool is bad for the whole tested year, the hypothesis about good model still need not to be rejected.

5. Correlation

This section describes how correlation was incorporated into TLA approach and normal test. Without correlation, *BDR* is limited by estimated *PD*. If this limit is exceeded, exception is generated. When correlation is taken into account, the limit is changed according to the relationship among *PD*, correlation and confidence interval. This relationship was described by one factor model which was used e.g. by Blochwitz et al. (2005) or observed by Castermans et al. (2007) and which is as follows:

$$PD_{correl} = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right). \tag{5}$$

PD_{corel} is the new limit based on which exception will be generated, ρ is the correlation, confidence interval α is defined by directive as 99%, $\Phi(x)$ is the cumulative standard normal distribution and $\Phi(x)^{-1}$ its inverse. The higher correlation, the less number of exceptions to appear. The value of correlation is prescribed by the Directive and is based on the portfolio type and product type.

When the correlation shifts limit for exception to the higher values, there naturally arises question of whether one should believe in the new number of exceptions and hence in the new results of *PD* backtest. Stein (2003) suggested approach which computes lower bound for the necessary number of data to conclude that the results are significant. The equation is following:

$$n \geq \frac{PD \cdot (1 - PD)}{(PD_{corr} - PD)^2} \left(\Phi^{-1}(1 - c/2)\right)^2, \qquad (6)$$

Where n is the minimal required number of accounts in each tested rating class. The significance of the results is in this paper expressed as ratio of real number of accounts and lower bound and denoted as *SIG*.

6. Results

6.1 TLA and Normal Test on Product Level

Results on two credit retail products can be seen in the Table 4. The product level was applied on the mean number of exceptions per pool in each individual month for TLA, hence having number of observations equal to 12. The normal test was applied on all observations, given by multiplication of number of pools and number of months (the exact number of observations for each product can bee also seen in Table 4.).

Table 4 Results on Product Level

	without correlation						with correlation				
	TLA			normal test			*TLA*		normal test		
prod.	**e**	**zone**	**#**	**unbiased**	**biased**	**#**	**e**	**zone**	**unbiased**	**biased**	***SIG***
A	0	**g**	12	**ok**	ok	240	0	**g**	**ok**	ok	125.25
B	6	**r**	12	**ok**	ok	192	0	**g**	**ok**	ok	18.00

Source: author's calculation

Without correlation, *TLA* approach assigned product A with its 0 mean exception per pool into the green zone and product B with its 6 exceptions per pool into the red zone. With correlation, both products are in the green zone. Normal test did not reject the hypothesis about good model regardless of the correlation usage.

6.2 TLA and Normal Test on Pool Level

Result of the backtest on the pool level of product A can be seen in Table 5.

Table 5 Results of backtest on pool level for product A

	without correlation						with correlation				
	TLA		normal test				*TLA*		normal test		
pool	*e*	zone	#	unbiased	biased	#	*e*	zone	unbiased	biased	*SIG*
0	0	g	12	**ok**	ok	12	0	g	**ok**	ok	71.62
1	1	y	12	**ok**	ok	12	0	g	**ok**	ok	129.99
2	6	r	12	**ok**	ok	12	0	g	**ok**	ok	164.43
3	7	r	12	**ok**	ok	12	0	g	**ok**	ok	263.35
4	1	y	12	**ok**	ok	12	0	g	**ok**	ok	342.18
5	0	g	12	**ok**	ok	12	0	g	**ok**	ok	446.91
6	0	g	12	**ok**	ok	12	0	g	**ok**	ok	251.25
7	0	g	12	**ok**	ok	12	0	g	**ok**	ok	295.53
8	0	g	12	**ok**	ok	12	0	g	**ok**	ok	240.99
9	0	g	12	**ok**	ok	12	0	g	**ok**	ok	421.11
10	0	g	12	**ok**	ok	12	0	g	**ok**	ok	254.15
11	0	g	12	**ok**	ok	12	0	g	**ok**	ok	188.00
12	0	g	12	**ok**	ok	12	0	g	**ok**	ok	87.31
13	0	g	12	**ok**	ok	12	0	g	**ok**	ok	142.09
14	0	g	12	**ok**	ok	12	0	g	**ok**	ok	70.85
15	0	g	12	**ok**	ok	12	0	g	**ok**	ok	110.18
16	0	g	12	**ok**	ok	12	0	g	**ok**	ok	312.22
17	0	g	12	**ok**	ok	12	0	g	**ok**	ok	73.06
18	0	g	12	**ok**	ok	12	0	g	**ok**	ok	5.26
19	1	y	12	**ok**	ok	12	0	g	**ok**	ok	**0.93**

Source: author's calculation

It can be seen from Table 5 that without correlation, product A has 3 pools in yellow zone and 2 pools in red zone. Inducing correlation, all pools appear in the green zone. Regardless of the correlation, the normal test did not reject the hypothesis about good model. Pool number 19 has not sufficient number of data to be sure about the green zone under correlation, which is indicated by the *SIG* less than 1.

Result of the backtest on the pool level of product B can be seen in Table 6. This table shows that product B is much worse without correlation than product A. It has also 3 pools in yellow zone, however 10 pools in red zone. Five of the 10 red pools are also "rejected" by the normal test using unbiased estimate of variance. After using correlation, only one pool remained not to be in the green zone (pool 14 in the yellow zone). Although pool 15 has sufficient number of observation according to the Stein's approach, *SIG* is only slightly higher than 1.

Table 6 Results of backtest on pool level for product B

pool	without correlation						with correlation				
	TLA			normal test			*TLA*		normal test		
	e	zone	#	unbiased	biased	#	*e*	zone	unbiased	biased	*SIG*
0	0	g	12	**ok**	ok	12	0	g	**ok**	ok	10.21
1	11	r	12	**reject**	reject	12	0	g	**ok**	ok	9.88
2	9	r	12	**reject**	reject	12	0	g	**ok**	ok	15.13
3	12	r	12	**reject**	reject	12	0	g	**ok**	ok	22.29
4	9	r	12	**ok**	ok	12	0	g	**ok**	ok	21.28
5	5	r	12	**ok**	ok	12	0	g	**ok**	ok	22.85
6	9	r	12	**reject**	ok	12	0	g	**ok**	ok	26.25
7	5	r	12	**ok**	ok	12	0	g	**ok**	ok	30.97
8	7	r	12	**ok**	ok	12	0	g	**ok**	ok	43.92
9	1	y	12	**ok**	ok	12	0	g	**ok**	ok	53.02
10	0	g	12	**ok**	ok	12	0	g	**ok**	ok	94.87
11	9	r	12	**ok**	ok	12	0	g	**ok**	ok	34.13
12	0	g	12	**ok**	ok	12	0	g	**ok**	ok	119.92
13	11	r	12	**reject**	reject	12	0	g	**ok**	ok	174.22
14	1	y	12	**ok**	ok	12	1	y	**ok**	ok	8.74
15	1	y	12	**ok**	ok	12	0	g	**ok**	ok	1.92

Source: author's calculation

7. Discussion

It may look from this paper that limit of *PD*, which is induced by taking correlation into account, might be the value toward which the next estimate of *PD* should be pulled. However, the correlation is already taken into account at the higher level of computing capital requirement according to equations prescribed by the Directive, hence there is no need to influence in this way original estimated *PD*.

There is need to know, that incorporating correlation based on the one factor model assumes infinite granular rating grades. Hence, effect of finite pool size should be also taken into account in future development of the method.

Basel II suggests different correlation for different products, however, the same for different countries. Some approaches to compute own correlations were mentioned by Ching et al. (2006). It also could by one of the issues that will be solved in Basel III.

While observing backtesting results by the bank's business experts or national regulator, there is a necessity to understand the way in which backtested default rate was computed. There can be several definitions of default rate inside individual banks, hence using these definitions, higher or lower backtested rates can be derived and different backtested results achieved. It is important to realize this before imposing multiplication factor to the *PD* estimate.

8. Conclusions

This work takes the Directive suggestions for *PD* backtest derived on market risk as basis and shows possible way in which *PD* backtest can be implemented for credit risk. At the beginning or this work, the main obstacle seemed to be number of data. It was shown that although only 12 values per year for each pool are available (meanwhile in market risk, the number of values is 250), the results obtained on such data set are also significant. The only exception is one or two pools in the portfolio example. The solution is to make smaller number of pools and hence higher number of individual accounts in each pool. This solution should be approved and implemented before next *PD* backtest, which is targeted on quarterly bases.

On the product level, each product is in the green zone and the normal test does not reject hypothesis about good model. On the pool level, all pools are in the green zone except one pool of product B, which is in yellow zone. Not sufficiently higher number of data is available for this pool, hence for the next backtest, the smaller number of pools is to be derived.

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