ESTIMATION OF FUTURE PD OF FINANCIAL INSTITUTIONS ON THE BASIS OF SCORING MODEL[1]

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Abstract

One of the most important tasks within the risk management is coherent determination of probability of default (PD). There have been proposed several distinct approaches to PD estimation, eg. on the basis of market prices (implied PD) or statistical models, involving a set of qualitative and quantitative measures. However, it is no less important to be able to estimate the evolution of the PD in the future. Our task in this paper is to estimate the probability distribution of a future PD for three Czech banks. The initial PD is calculated on the basis of a scoring model, developed recently for US banks by one of the coauthors by using linear discriminant analysis. Next, we sample randomly the values of particular indicators and estimate the PDs' distribution. We assume that the indicators are distributed according to a multidimensional subordinated Lévy model. We also present the joint probability of high PD's. Although all banks are relatively healthy, there is still high chance that "a financial crisis" will occure, at least in terms of probability. Moreover, high sensitivity to model selection is documented.

Keywords: *credit-scoring models; probability of default; multidimensional subordinated Lévy model*

JEL codes: *C01, C51, G17*

1. Introduction

The probability of default (PD), ie. the probability, that a given entity will not be able or willing to meet its obligations, is a crucial input factor of credit risk modeling and measuring. Designing of efficient techniques to its estimation is therefore in the spotlight of many research units.

A common approach to PD estimation is to apply a statistical model, derived by a suitable econometric method applied to collected financial (quantitative) data or measures of

[1] The research is due to the support provided by GAČR (Czech Science Foundation – Grantová Agentura České Republiky) under the project No. 402/08/1237. All support is greatly acknowledged.

rather personal nature (qualitative features). A family of these models is generally referred to as *credit scoring models*, including the one, introduced in the seminal paper of Altman (1968) – see also Altman and Saunders (1998). Alternatively, when the entity has issued stocks or bonds, which are tradable and liquid, the PD can be estimated from the market prices, see e.g. Merton (1974) for the first application of the structural approach (stock price). However, following this approach we obtain the risk-neutral estimation of PD.

The vast majority of already proposed credit scoring models were derived on a sample of non-financial institutions, mainly due to the fact that defaults of financial institutions occur relatively scarcely and not all the data are publicly available. Nevertheless, there were several more or less sufficient attempts to identify the key factors for healthy financial institutions, originating from financial statements, see e.g. Martin (1977) or Peresetsky and Karminsky (2008) and references therein. Even more recently, Gurný and Gurný (2009a,b) proposed a two stage model, ie. either default or non-default stage, on the basis of publicly available data of banks mainly from the US, henceforth the GaG model. The model was latter applied in order to assess the creditworthiness of several Czech banks.

Credit scoring models provide us a static estimation of probability of default, usually for a one year horizon. In this paper, however, the task is to go even further and estimate the probability distribution of future PD for several Czech banks assuming that the significant financial indicators follows multidimensional subordinated Lévy models. In order to determine the PD, the revised GaG model is used.

We proceed as follows. In the following section, GaG model is introduced. Next, in Section 3 the multidimensional subordinated Lévy model is defined. Finally, in Section 4 we estimate the future distribution of banks' PD on the basis of quarterly collected financial statements.

2. Credit Scoring Models

Credit scoring models are statistical models derived by means of econometric methods, such as regression analysis, discriminant analysis, logit and probit models, or even neural networks or panel models – see any econometric textbook or credit risk handbook, such as Green (2008) or Engelmann and Rauhmeier (2006) – that allow us to classify borrowers according to their probability of default and potentially assign a rating category.

Probably the best known credit scoring model is the one introduced by Altman (1968) for US listed companies on the basis of linear discriminant analysis. Obviously, this model has been up-dated many times, including adjustments for different sectors or regions. Within the Czech Republic, the best known models are so called IN models (see e.g. Neumaierová and Neumaier, 2002). Recently, Jakubík and Teplý (2008) formulated an alternative model for the Czech corporate sector on the basis of logit analysis. However, due to our knowledge, there exists no model exclusively for financial institutions of Central Europe.

In the following lines we will briefly describe the linear discriminant analysis. Then, we will present the most recent model of Gurný and Gurný (2009b), the model that was proposed for US banks on the basis of discriminant analysis.

2.1 Linear Discriminant Analysis

Linear discriminant analysis targets on detection of a linear discriminant function that would allow us a successful separation of the sample into two groups according to a specific set of features.

A basic principal is to maximize the difference between the two groups, while the differences among particular members of the same group are minimized. Within credit risk models, one group consists of good borrowers (non-defaulted – *G*), while the other includes bad ones (already defaulted – *B*). The differences are measured by means of the discriminant variable – *score Z*. For a given borrower *i*, we calculate the score as follows:

$$Z_i = \sum_{j=1}^{n} \gamma_j x_{i,j} \ , \tag{1}$$

where x denotes a given feature (usually financial measure, e.g. obtained from balance sheet) and γ is its coefficient within the estimated model. These coefficients can be obtained by

$$\gamma = \Sigma^{-1}(x_G - x_B)\,, \tag{2}$$

where x_A and x_B are vectors containing the mean values of n variables (ie. x_i for $i = 1,\dots n$) for the group G and B, respectively, and Σ is the variance-covariance matrix of these variables and can be estimated as the "average" of respective matrices for each group, weighted by the sample size (n_G and n_B):

$$\Sigma = \frac{n_G - 1}{n_G + n_B - 2}\Sigma_G + \frac{n_G - 1}{n_G + n_B - 2}\Sigma_B \,.$$

Finally, the score Z can be transformed into the probability of default of a given entity i:

$$PD_i = \frac{1}{1 + \frac{1-\pi_B}{\pi_B} e^{z_i - \alpha}} . \quad (4)$$

Here, α is estimated cut-off point:

$$\alpha = \frac{1}{2}\gamma'(x_G - x_B) . \quad (5)$$

and π_B represents the prior probability of default, which depends on the general characteristic of the market (or borrower's portfolio). For more detailed description of this approach, including illustrative examples, see e.g. Resti and Sironi (2007) or any other text book on credit risk modeling and measuring.

2.2 GaG Models

Initially, Gurný and Gurný (2009a) worked with the sample of 14 banks – eight banks were recognized as good and six as bad, ie. they defaulted or were very close to the default state during 2008. Obviously, the authors used financial statements from the previous year. The proposed model to calculate the score was:

$$z_i = -0{,}8x_{2,i} - 6{,}2x_{4,i} + 0{,}0002x_{6,i} - 18x_{7,i} + 12x_{11,i} , \quad (6)$$

where x_2, x_4, x_6, x_7, and x_{11} state for LTA (*Log total assets*), YAEA (*Yield on average interest earning assets*), NIM (*Net interest margin*), ROAA (*Return on average assets*), and PE OI (*Personal expenses on operational income*), respectively. Since the resulting model was not very successful, as given by very low level of Wilk's lambda (0.304), the authors decided to extend the sample of both defaulted and non-defaulted banks.

In Gurný and Gurný (2009b), more successful model was proposed, with Wilk's lambda close to 0.73. Into this model, 18 good and 18 bad banks were involved. The model looks as follows:

$$z_i = 178x_{1,i} - 120x_{3,i} + 159x_{4,i} - 61x_{10,i} \quad (7)$$

where x_1, x_3, x_4, and x_{10} denotes YAEA, NIM, ROAA, and PL GL (*Problem loans on gross loans*), respectively. Thus, three profitability and one asset quality ratios were identified. The cut off ratio α is 3.28.

In Table 1, z score and estimated probability of default for all banks from the sample are provided. The model provide almost zero probability of default for almost all banks from group G – only two banks exhibit higher PD than 10%, and only five should defaulted with probability higher than 1%. By contrast, the model is not so successful when determining bad entities. For example, there are several banks with PD close to or even bellow 50%, although they already defaulted. Since investors are generally risk averse, any bank with higher PD than, say, 10%, should be carefully monitored.

Table 1 Z-score and PD estimation for all banks of the sample

Non-default banks (Group G)	z_i	*PD*	Default banks (Group B)	z_i	*PD*
Bank of America Corporation	9.680	0.2%	Aliance Bank	0.506	94.1%
JPMorgan Chase & Co.	11.794	0.0%	Bank of Clarke County	3.143	53.4%
M&T Bank Corporation	5.643	8.6%	BankUnited	3.035	56.1%
National City Corporation	5.017	15.0%	Citizens Community Bank	2.295	72.8%
PNC Bank	9.184	0.3%	Michigan Heritage Bancorp	–5.343	100.0%
SunTrust Bank	8.838	0.4%	National Bank of Commerce	–1.364	99.0%
Wells Fargo & Company	4.357	25.4%	Omni Financial Services	–1.832	99.4%
Zionsbancorporation	8.487	0.5%	First Bank of Idaho	–1.425	99.1%
State Street Corporation	11.623	0.0%	Citizens National Bank	–1.015	98.7%
Bank of NY Mellon Corporation	9.565	0.2%	Silverton Bank	–3.284	99.9%
US Bancorp	5.551	9.3%	American Stearling Bank	–4.827	100.0%
Regions Bank	9.463	0.2%	First Priority Bank	–8.479	100.0%
Capital One Financial Corporation	12.167	0.0%	Douglass National Bank	–8.982	100.0%
KeyCorp	9.174	0.3%	First National Bank	3.688	39.9%
Marshall & Isley Corp	10.086	0.1%	Hume Bank	1.749	82.2%
Colonial BancGroup	8.932	0.3%	First Heritage Bank	–6.544	100.0%
Northern Trust Corporation	5.690	8.2%	Bank of Georgia in Commerce	–1.701	99.3%
Webster Financial Corporation	10.781	0.1%	Great Basin Bank of Nevada	–7.593	100.0%
mean values	**8.668**	**3.8%**	**mean values**	**–2.110**	**88.5%**

Source: Gurný and Gurný (2009b)

3. Multidimensional Subordinated Lévy Models

The first focus at Lévy models with jumps goes back to 1930's. The most recent and complete monographs on the theory behind and/or application of Lévy models are Kyprianou et al. (2005), Applebaum (2004), Cont and Tankov (2004), Barndorff-Nielsen et al. (2001) and Bertoin (1998). However, a subordinated Lévy model, a rather non-standard definition of Lévy models as time changed Brownian motions, goes back to Mandelbrot and Taylor (1967), Clark (1973) or even Bochner (1949). In this section, we first describe the marginal distributions of subordinated Lévy models. Then, we will show, how to obtain multidimensional distribution from marginal distributions by means of copula functions.[2]

[2] For an alternative approach to building up multidimensional Lévy models, see e.g. Tichý (2008a).

3.1 Marginal Distribution

Generally, a Lévy process is a stochastic process, which is zero at origin, its path in time is right-continuous with left limits and its main property is that it is of independent and stationary increments. Another common feature is a so called stochastic continuity. Moreover, the related probability distribution must be infinitely divisible. The crucial theorem is the Lévy-Khintchine formula:

$$\Phi(u) = \mathrm{i}\gamma u - \frac{1}{2}\sigma^2 u^2 + \int_{-\infty}^{\infty} \left(\exp(\mathrm{i}ux) - 1 - \mathrm{i}ux\mathbb{I}_{|x|<1}\right)\nu(\mathrm{d}x). \tag{8}$$

For a given infinitely divisible distribution, we can define the triplet of Lévy characteristics,

$$\{\gamma, \sigma^2, \nu(\mathrm{d}x)\}.$$

The former two define the drift of the process (deterministic part) and its diffusion. The latter is a Lévy measure. If it can be formulated as $\nu(\mathrm{d}x) = u(x)\mathrm{d}x$, it is a Lévy density. It is similar to the probability density, with the exception that it need not be integrable and zero at origin.

Let X be a Brownian motion. If we replace standard time t in Brownian motion X,

$$X(t; \mu, \sigma) = \mu dt + \sigma\mathcal{Z}(t), \tag{9}$$

by its suitable function $\ell(t)$ as follows:

$$X(\ell(t); \theta, \vartheta) = \theta\ell(t) + \vartheta\mathcal{Z}\left(\ell(t)\right) = \theta\ell(t) + \vartheta\varepsilon\sqrt{\ell(t)}, \tag{10}$$

we get a subordinated Lévy model.

Due to the simplicity (tempered stable subordinators with known density function in the closed form), the most suitable candidates for the function $\ell(t)$ seem to be either the variance gamma model – the overall process is driven by a gamma process from the gamma distribution with shape a and scale b depending solely on variance κ, $G[a, b]$, or normal inverse Gaussian model – the subordinator is given by an inverse Gaussian process based on the inverse Gaussian distribution, $IG[a, b]$. [3]

[3] For exact procedure how to simulate both models by alternative Monte Carlo simulation technice, see e.g. Tichý (2008b).

3.2 Copula functions

A useful tool for dependency modeling are the copula functions,[4] ie. the projection of the dependency among particular distribution functions into $[0,1]$,

$$\mathcal{C}: [0,1]^n \to [0,1] \text{ on } \mathbb{R}^n,\ n \in \{2,3,\dots\}. \tag{13}$$

Actually, any copula function can be regarded as a multidimensional distribution function with marginals in the form of standardized uniform distribution.

For simplicity, assume two potentially dependent random variables with marginal distribution functions F_X, F_Y and joint distribution function $F_{X,Y}$. Then, following the Sklar's theorem:

$$F_{X,Y}(x,y) = \mathcal{C}(F_X(x), F_Y(y)). \tag{14}$$

If both F_X, F_Y are continuous, a copula function $\mathcal{C}$ is unique. Sklar's theorem implies also an inverse relation,

$$\mathcal{C}(u,v) = \mathrm{F}_{X,Y}\left(F_X^{-1}(u), F_Y^{-1}(v)\right). \tag{15}$$

Formulation (14) above should be understood such that the joint distribution function gives us two distinct information: (i) marginal distribution of random variables, (ii) dependency function of distributions. Hence, while the former is given by $F_X(x)$ and $F_Y(y)$, a copula function specifies the dependency, nothing less, nothing more. That is, only when we put both information together, we have sufficient knowledge about the pair of random variables X, Y.

Assuming that the marginal distribution functions of random variables are already known, the only further think we need to know to model the overall evolution is an appropriate copula function. With some simplification, we can distinguish copulas in the form of elliptical distributions and copulas from the Archimedean family. The main difference between these two forms is given by the ways of construction and estimation. While for the latter the primary assumption is to define the generator function, for the former the knowledge of related joint distribution function (e.g. Gaussian, Student-t, etc.) is sufficient.

3.3 Parameter Estimation of Multidimensional Subordinated Lévy Models

There exist three main approaches to parameter estimation for copula function based dependency modeling: exact maximum likelihood method (EMLM), inference for margins

[4] In this paper, we restricted ourselves to ordinary copula functions. Basic reference for the theory of copula functions is Nelsen (2006), while Rank (2007) and Cherubini et al. (2004) target mainly on the application issues in finance. Alternatively, Lévy processes can be coupled on the basis of Lévy measures by Lévy copula functions. However, this approach is not necessary in our case.

(IFM), and canonical maximum likelihood (CML). While for the former all parameters are estimated within one step, which might be very time consuming, mainly for high dimensional problems or complicated marginal distributions, the latter two methods are based on estimating the parameters for the marginal distribution and parameters for the copula function separately. While assuming IFM, marginal distributions are estimated in the first step and the copula function in the second one, for CML instead of parametric margins empirical distributions are used. On more details see any of the empirically oriented literature such as Cherubini et al. (2004).

In this paper we will assume IFM approach. In order to estimate the parameters of marginal distributions, generalized method of moments will be used (Table 2).

Table 2 Generalized method of moments for VG and NIG models

Moments	**VG**	**NIG**
Mean	θ	θ
Variance	$\vartheta^2 + \nu\theta^2$	$\vartheta^2 + \nu\theta^2$
Skewness	$\theta\nu\left(3\vartheta^2 + 2\nu\theta^2\right)\left(\vartheta^2 + \nu\theta^2\right)^{-3/2}$	$3\theta\nu\left(\vartheta^2 + \nu\theta^2\right)^{-1/2}$
Kurtosis	$3\left(1 + 2\nu - \frac{\nu\vartheta^4}{\left(\vartheta^2 + \nu\theta^2\right)^2}\right)$	$3\left(\frac{\theta^2\nu(1+5\nu) + \vartheta^2(1+\nu)}{\vartheta^2 + \nu\theta^2}\right)$

4. Estimation of PD Distribution

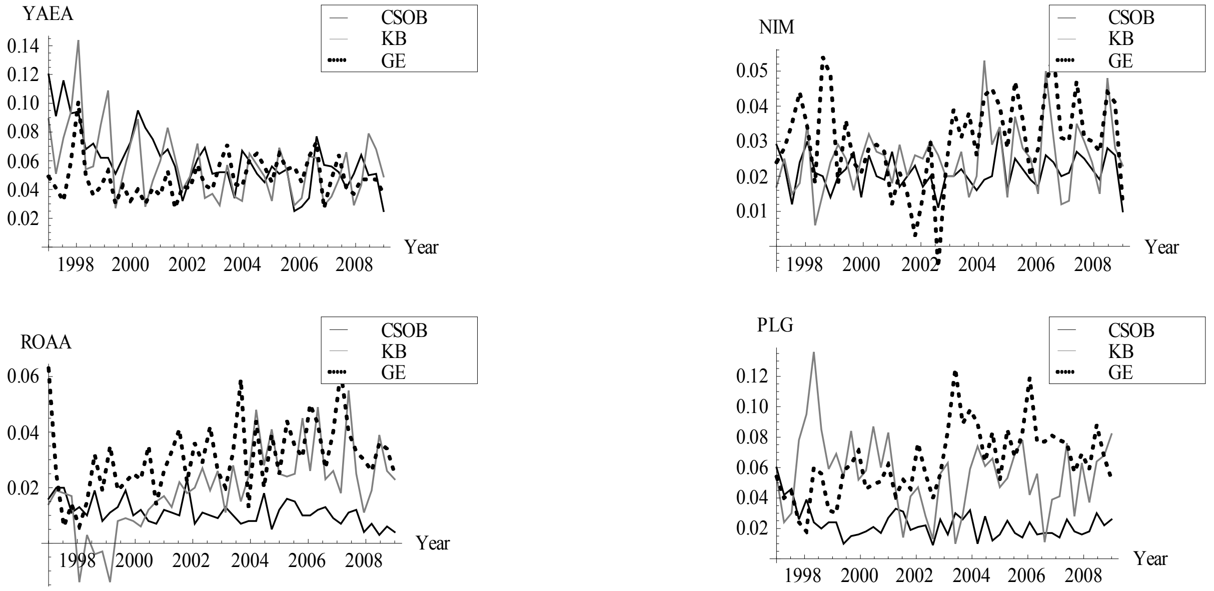
In this section we will proceed to the main task of the paper. First, we describe the data set – financial indicators. Then, we estimate their future marginal distribution. Finally, we connect marginal distributions together by a given copula function and estimate the distribution of future PD as based on model (7).

4.1 Data

We collected four financial indicators of three banks on the quarterly basis over last ten years. In particular, we study ČSOB (*Československá obchodní banka*), KB (*Komerční banka*), and GE (*GE Money Bank*) on the basis of financial indicators indentified by Gurný and Gurný (2009b) for the set of (mainly) US banks. In order to estimate PD, e.g. to transform score z into the probability, see formula (4), we use $\pi = 0.1$.

The indicators, which have been already identified as significant are Interest income on Average interest earning assets (*YAEA*), Net interest margin (*NIM*), Return on average assets (*ROAA*) and Problem loans on gross loans (*PL GL*). We present their evolution during last ten years in Figure 1. Moreover, basic descriptive statistics are depicted in Table 6 in Appendix.

Figure 1 Evolution of financial indicators over 1997–2009



Source: Author's calculation

It is also important to know the dependency among particular marginal distributions. Despite its well-known drawbacks, the measure of linear correlation is still the most popular approach how to express the dependency, see Table 3.

Table 3 Pearson correlation of financial indicators over 1997–2009

		CSOB				*KB*				*GE*			
		YAEA	**NIM**	**ROAA**	**PL GL**	**YAEA**	**NIM**	**ROAA**	**PL GL**	**YAEA**	**NIM**	**ROAA**	**PL GL**
CSOB	**YAEA**	1.000	0.087	0.306	0.530	0.486	–0.156	–0.435	0.023	0.024	0.060	–0.278	–0.502
	NIM	0.087	1.000	0.135	0.092	0.300	0.349	0.155	0.215	0.416	0.353	0.150	0.065
	ROAA	0.306	0.135	1.000	0.270	0.149	–0.127	–0.193	–0.090	0.097	0.040	–0.227	–0.266
	PL GL	0.530	0.092	0.270	1.000	0.465	–0.146	–0.171	–0.061	0.115	–0.017	–0.035	–0.205
KB	**YAEA**	0.486	0.300	0.149	0.465	1.000	0.338	–0.364	0.157	0.468	0.177	–0.194	–0.476
	NIM	–0.156	0.349	–0.127	–0.146	0.338	1.000	0.442	–0.005	0.449	0.322	0.102	0.103
	ROAA	–0.435	0.155	–0.193	–0.171	–0.364	0.442	1.000	–0.192	0.132	0.184	0.340	0.553
	PL GL	0.023	0.215	–0.090	–0.061	0.157	–0.005	–0.192	1.000	0.120	0.085	–0.279	–0.180
GE	**YAEA**	0.024	0.416	0.097	0.115	0.468	0.449	0.132	0.120	1.000	0.380	–0.056	0.085
	NIM	0.060	0.353	0.040	–0.017	0.177	0.322	0.184	0.085	0.380	1.000	0.015	0.218
	ROAA	–0.278	0.150	–0.227	–0.035	–0.194	0.102	0.340	–0.279	–0.056	0.015	1.000	0.421
	PL GL	–0.502	0.065	–0.266	–0.205	–0.476	0.103	0.553	–0.180	0.085	0.218	0.421	1.000

Source: Author's calculation

In Figure 1 we could observe that almost all indicators were very instable in time. It also results into high variability of PD in time. It is very interesting, that according to the model, e.g. KB was very close to default stage several times during the period, mainly due to negative values of ROAA and high level of problematic loans. By contrast, CSOB performed relatively well in time. If we take into account only the last values, the default probabilities for particular banks should be 0.23, 0.031, and 0.008, respectively.

4.2 Marginal Distribution

In order to get the future distribution of particular financial indicators x_i, we will assume n = 100 000 independent scenarios of plain Monte Carlo simulation[5] for (i) Gaussian distribution, $x_i \in N[0,1]$, (ii) variance gamma distribution, $x_i \in VG(\nu;\theta,\vartheta)$, and ($iii$) normal inverse Gaussian distribution $x_i \in NIG(\nu;\theta,\vartheta)$. It is generally recommended to use maximal likelihood approach for estimation of the model parameters. However, since our data set consist of only few values, we will follow the generalized method of moments.

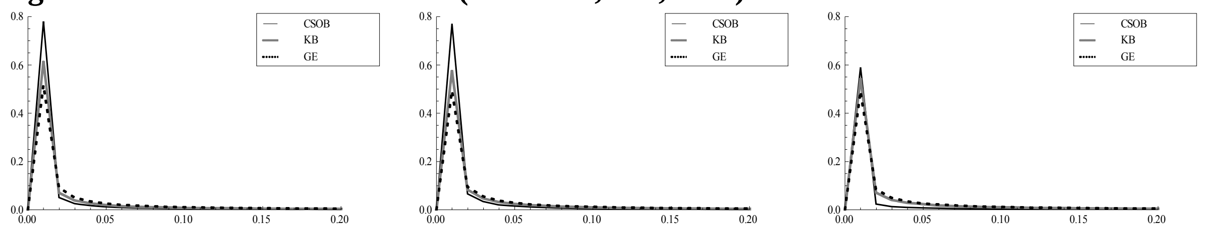
In this paper we are interested solely in the distribution of PD for the next time moment. We therefore sample just once from the estimated distribution to get one scenario, ie. the future value of a given indicator x_i.

Next point to consider is that several financial indicators are on the percentage basis and cannot go outside the range [0,1]. We therefore set constrains on admissible values and transform them to preserve the empirical features of the distribution.

4.3 Linear Dependency among Factors – Gaussian Copula

In order to estimate the probability of default of any of the banks, the dependency among particular indicators must be taken into account. We first consider the Gaussian copula function, ie. the inputs are marginal distributions and empirical correlation matrix. The resulting distribution of PD is depicted in Figure 2.

Figure 2 Estimation of future PD (Gaussian, VG, NIG)



Source: Author's calculation

[5] See Tichý (2008b) for the efficiency examination with respect to subordinate Lévy models.

Another interesting issue is, what is the probability that PDs for all banks will increase over 10%, 20% or even 50%? We provide the results in Table 4. For example, with probability 0.3, the PD of all three banks will be at least 10% within VG model.

Table 4 Joint probability of PDs, Gaussian copula approach

PD	Gauss	VG	NIG
10%	0.00761	0.00337	0.03262
20%	0.00289	0.00072	0.01711
50%	0.00052	0.00002	0.00419

Source: Author's calculation

4.4 Stressed Tails of Factors' Distribution – Student Copula

In preceding subsection, Gaussian copula was assumed, ie. the dependency among particular indicators was linear. Now, we will try to stress the dependency in tails, that is, if one indicator is very good (bad), the same is true for the others, irrespective the bank. For that reason, Student copula function is applied. In order to estimate the number of degrees of freedom (*df*), the approach of IFM is used – we get $df = 5$.

The graphical presentation of PD distribution is not significantly different to the Gaussian copula approach, independently on the model/bank. However, the joint probabilities are quite different, see Table 5.

Table 5 Joint probability of PDs, Student copula approach

PD	Gauss	VG	NIG
10%	0.00875	0.00220	0.02664
20%	0.00385	0.00046	0.01301
50%	0.00100	0.00002	0.00291

Source: Author's calculation

5. Conclusion

Estimation of future evolution of PD of debtors should be regarded as a very important task. In this paper we have tried to estimate the distribution of a next stage PD of three Czech banks when the PD is supposed to be determined by the evolution of financial indicators through credit scoring model of Gurný and Gurný (2009b), proposed recently for US banks.

We have assumed three candidates for the evolution of financial indicators, Gaussian distribution, VG distribution and NIG distribution. Regardless the model, as the most risky bank, GE Money Bank should be considered. Next, CSOB is slightly less risky than KB. It is interesting to note, that the results of VG model are more close to Gaussian model than to NIG model, which is quite surprising. Usually, assuming any other financial application, see eg. Tichý (2008a), the results of VG model are not very different to the ones obtained for NIG model.

In further research, however, higher attention should be paid to the quality of marginal distribution modeling, ie. the probability distribution of particular financial indicators. Moreover, it would be also interesting to examine the effect of copula functions from Archimedean family on the joint distribution of PD's in banking sector as a whole.

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Table 6 Descriptive statistics of the distribution of particular financial indicators

		YAEA	NIM	ROAA	PL GL
CSOB	*Mean*	0.0608	0.0212	0.0110	0.0230
	St.dev.	0.0209	0.0050	0.0044	0.0097
	Skewness	0.7863	0.0336	0.6608	1.5164
	Kurtosis	3.9945	3.1842	3.2969	6.4973
KB	*Mean*	0.0569	0.0251	0.0197	0.0558
	St.dev.	0.0240	0.0096	0.0143	0.0246
	Skewness	1.2327	0.8802	0.0880	0.4051
	Kurtosis	5.4396	4.3398	3.8364	4.2010
GE	*Mean*	0.0480	0.0303	0.0308	0.0639
	St.dev.	0.0139	0.0129	0.0129	0.0221
	Skewness	1.2968	-0.3647	0.4792	0.4736
	Kurtosis	6.2185	3.3180	3.4358	3.6464