A Financial Application of Multivariate Stochastic Orderings Consistent with Preferences

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Abstract
In this paper we discuss and evaluate some possible applications of multivariate stochastic orderings consistent with the investors' preferences. Thus, starting by a recent classification of stochastic orderings consistent with preferences, we show how risk/variability of multivariate measures are used to obtain non dominated choices in some financial problems. Then we examine orderings that satisfy an opportune identity property and the basic rules of the theory of integral stochastic orders. In this framework we propose a possible financial application where multivariate preferences must be applied to determine the dominance of a market respect to another one. In particular we propose a first ex-post empirical comparison to evaluate the possible dominance among the US stock market, the German stock market and the UK stock exchange market.

Keywords: Multivariate Stochastic Orderings, Financial Markets.
JEL codes: G11

1. Introduction

In this paper we introduce multivariate orderings consistent with investors' preferences and we classify them, distinguishing several categories associated with different classes of investors. Moreover, we show how we can use multivariate risk measures and orderings consistent with some preferences to determine dominant sectors, markets in different financial contexts. We are interested in the economic use of probability functionals to optimize choices for a given order of investors' preferences. For this reason we first define the dominance among financial markets and we propose a first empirical application of multivariate orderings in this context.

Thus we first generalize the concept of univariate FORS orderings, risk and reward measures in the multivariate framework (Ortobelli et al. 2008, 2009, 2013). FORS probability functionals and orderings generalize those found in the literature (Shaked, M., and Shanthikumar, 1993) and are strictly related to the theory of choice under uncertainty and to theory of probability functionals and metrics (Rachev, 1991 and Stoyanov et al. 2008). While the new orderings serve to further characterize and specify the investors' choices/preferences, the new risk measures should be used either to minimize the risk or to minimize its distance from a given benchmark. Secondly we propose an empirical comparison to evaluate the possible dominance among different financial markets.

The paper is organized as follows. In Section 2 we introduce multivariate FORS orderings. Section 3 introduces a preliminary empirical analysis while the last section summarizes some other possible financial applications.

2. FORS measures and orderings

In this section we introduce FORS multivariate measures and orderings. Recall that the most important property that characterizes any probability functional associated with a choice problem is the consistency with a stochastic order. In terms of probability functionals, the consistency is defined as: *X* dominates *Y* with respect to a given order of preferences $\succ$ implies $\mu(X,Z) \le \mu(Y,Z)$ for a fixed arbitrary benchmark Z (where $X,Y,Z \in \Lambda$ that is a non-empty space of real valued random variables defined on $(\Omega, \Im, P)$). Since an univariate FORS measure induced by of preferences $\succ$ is any probability functional $\mu : \Lambda \times \Lambda \to R$ that is consistent with a given order of preferences $\succ$ we can similarly define multivariate FORS measures.

Definition 1 *We call FORS measure induced by a preference order* $\succ$ *any probability functional* $\mu : \Lambda \times \Lambda \to R^s$ *(where* Λ *a non-empty set of real-valued n-dimensional random vectors defined on the probability space (* $\Omega, \Im, P$ *)) that is consistent with a given order of preferences* $\succ$ *(that is, if* X *dominates* Y *with respect to a given order of preferences* $\succ$ *implies* $\mu(X,Z) \le \mu(Y,Z)$ *for a fixed arbitrary benchmark* Z *where the vectorial inequality is considered for each component i.e.,* $\mu_i(X,Z) \le \mu_i(Y,Z)$ *for any i=1,...,s).*

As for the FORS measures we can easily extend the definition of multivariate FORS ordering developed in Ortobelli et al. 2008 and 2009.

Definition 2 *Let* $\rho_X : A \to \overline{R}^s$ *(with compact and convex* $A \subseteq \overline{R}^n$ *) be a bounded variation function, for every n-dimensional random vector* X *belonging to a given class* Λ*. Assume that* $\forall X,Y \in \Lambda, \rho_X = \rho_Y$ *, a.e. on* A *iff* $X \overset{d}{=} Y$*. If, for any fixed* $\lambda \in A$*,* $\rho_X(\lambda)$ *is a FORS measure induced by an ordering* $\succ$*, then we call FORS orderings induced by* $\succ$ *the following new class of orderings defined* $\forall X,Y \in \Lambda_\alpha = \left\{ X \in \Lambda \; : \; \left| \int_A \prod_{i=1}^{n} |t_i|^{\alpha_i - 1} d\rho_X(t_1,...,t_n) \right| < \infty \right\}$ *for every* $(\alpha_1,...,\alpha_n)$ *with* $\alpha_i \ge 1,$ *we say that* X *dominates* Y *in the sense* α *-FORS ordering induced by* $\succ$ *, in symbols:*

$$X \; FORS_{\succ,\alpha} Y \text{ if and only if } \rho_{X,\alpha}(u) \le \rho_{Y,\alpha}(u) \;\; \forall u \in A$$

where

$$\rho_{X,\alpha}(u_1,...,u_n)=\begin{cases}\dfrac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i)}\int_{a_1}^{u_1}...\int_{a_n}^{u_n}\prod_{i=1}^{n}(u_i-t_i)^{\alpha_i-1}d\,\rho_X(t_1,...,t_n)\\ \rho_X(t_1,...,t_n)\quad if\quad \alpha_i=1;\ i=1,...,n\end{cases}\qquad (1)$$

and the integral is a vector applied for each component of the vector $d\,\rho_X=[d\rho_{(1)X},...,d\rho_{(s)X}]$ *whose components are the differential of the components of vector* $\rho_X=[\rho_{(1)X},...,\rho_{(s)X}]$.

This expression generalizes the one proposed by Petronio et al. 2013. Besides, we call ρ_X FORS measure associated with the FORS ordering of random vectors belonging to Λ. We say that ρ_X generates the FORS ordering.

Example 1: Consider the cumulative multivariate function associated with the vector *X*, $P_X(y)=P(X_1\le y_1,...,X_n\le y_n)=F_X(y_1,...,y_n)$. It generates the lower orthant FORS order (Shaked et al. 1993). So the measure associated to the α-FORS ordering is

$$F_X^{(\alpha)}(u)=\frac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i)}\int_{-\infty}^{u_1}...\int_{-\infty}^{u_n}\prod_{i=1}^{n}(u_i-t_i)^{\alpha_i-1}d\,F_X(t)=E\left(\prod_{i=1}^{n}(u_i-X_i)_+^{\alpha_i-1}\right)/\prod_{i=1}^{n}\Gamma(\alpha_i).\qquad (2)$$

2.1 Properties and characteristics of multivariate FORS orderings

As for the univariate FORS measures and orderings (Ortobelli et al. 2008 and 2009) using the properties of the fractional integral we can distinguish: *dual orderings*; *limited/unlimited orderings*, *survival orderings*; *risk/variability orderings*; *static/dynamic orderings*; *several levels of ordering*, etc. Given a FORS orderings the associated FORS measure ρ_X defined on the class of the random vectors Λ, then we get the following extensions and implications[1]:

1. For every $\alpha>>v\ge\mathbf{1}$ (i.e., $\alpha_i>v_i\ge 1$ with $i=1,...,n$) and $\alpha,v\in R^n$ $X\ \underset{\succ,v}{FORS}\ Y$ implies $X\ \underset{\succ,\alpha}{FORS}\ Y$ and we can write
$$\rho_{X,\alpha}(u_1,...,u_n)=\frac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i-v_i)}\int_{a_1}^{u_1}...\int_{a_n}^{u_n}\prod_{i=1}^{n}(u_i-t_i)^{\alpha_i-v_i-1}\rho_{X,v}(t_1,...,t_n)dt_1...dt_n$$
2. We define the survival FORS orderings saying that $X\ \underset{\succ,sur\alpha}{FORS}\ Y$ if and only if $\bar{\rho}_{X,\alpha}(t)\le\bar{\rho}_{Y,\alpha}(t)\ \ \forall t\in A\subseteq R^n$ where
$$\bar{\rho}_{X,\alpha}(u_1,...,u_n)=\begin{cases}\dfrac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i)}\int_{u_1}^{b_1}...\int_{u_n}^{b_n}\prod_{i=1}^{n}(t_i-u_i)^{\alpha_i-1}d\rho_X(t_1,...,t_n)\ if\ \alpha_i\ge 1\wedge\exists i\ :\ \alpha_i>1\\ -\bar{\rho}_X(u_1,...,u_n)\ \ if\ \ \alpha_i=1;\ \forall i=1,...,n\end{cases}$$

[1]For any $\alpha,\beta\in R^n$, we generally point out
$\alpha\ge\beta$ if $\alpha_i\ge\beta_i\ \forall i$
$\alpha>>\beta$ if $\alpha_i>\beta_i\ \forall i$
$\alpha>\beta$ if $\alpha_i\ge\beta_i\ \exists i\ :\ \alpha_i\ge\beta_i$

$b_i = \sup\{t_i / (t_1,...,t_i,...,t_n) \in A\}$ and for every $\alpha >> v \geq 1,\ \alpha, v \in R^n$, we obtain the analogous formula

$$\bar{\rho}_{X,\alpha}(u_1,...,u_n) = \frac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i - v_i)}\int_{u_1}^{b_1}...\int_{u_n}^{b_n}\prod_{i=1}^{n}(t_i - u_i)^{\alpha_i - v_i - 1}\bar{\rho}_{X,v}(t_1,...,t_n)dt_1...dt_n$$

3. We say that X dominates Y in the sense of α FORS variability (or uncertainty) ordering induced by $\succ$ (i.e. $X \underset{\succ, unc\alpha}{FORS} Y$) if and only if $\rho_{\pm X,\alpha}(u) \leq \rho_{\pm Y,\alpha}(u)\ \forall u \in A$ (i.e., $X \underset{\succ,\alpha}{FORS} Y$ and $-X \underset{\succ,\alpha}{FORS} -Y$).
4. Suppose $\|\rho_X(b)\| < \infty$, and/or $\|\rho_X(a)\| < \infty$ for every vector X belonging to Λ. Then we can extend ρ_X on all $\overline{R}^n$ assuming $\rho_X(u) = \rho_X(a)$, $\forall u \leq a$, i.e., $a_i = \inf\{t_i / (t_1,...,t_i,...,t_n) \in A\}$ and $\rho_X(u) = \rho_X(b)$, $\forall u \geq b$, i.e. $b_i = \sup\{t_i / (t_1,...,t_i,...,t_n) \in A\}$.
   Moreover, we say X unbounded α FORS dominates Y if and only if $\rho_{X,\alpha}(u) \leq \rho_{Y,\alpha}(u)$ for every $u \in \overline{R}^n$ where we define
   $$\rho_{X,\alpha}(u) = \frac{1}{\prod_{i=1}^{n}\Gamma(\alpha_i)}\int_{-\infty}^{u_1}...\int_{-\infty}^{u_n}\prod_{i=1}^{n}(u_i - t_i)^{\alpha_i - 1}d\rho_X(t)$$
   If ρ_X is monotone, then unbounded $\underset{\succ,\alpha}{FORS}$ order implies bounded $\underset{\succ,\alpha}{FORS}$ order.
5. For any monotone increasing FORS measure ρ_X associated with a FORS ordering, induced by $\succ$ the opposite of, the left inverse (that is supposed applied to each component of ρ_X), $-\rho_{X,}^{-1}(x)$ generates itself a FORS ordering.

In addition, the multivariate fractional integral can be seen as a particular multivariate transform, then starting by a multivariate FORS ordering we get different levels of multivariate FORS orderings.

Theorem 2 *Suppose* $|b_i| < +\infty\ (\forall i \leq n)$ *and* $\rho_X^{(1)}\ :\ A \to R^s$ *is a FORS1 measure associated with a FORS1 ordering* $\succ$ *defined on a class of random vectors* Λ. *If* $\rho_X^{(1)}$ *is a bounded and monotone function, then the probability functional* $\rho_X^{(2)}\ :\ [1,p]^n \to R^s$ *defined by* $\rho_X^{(2)}(u_1,...,u_n) = \rho_{X,u}^{(1)}(b_1,...,b_n)$ *points out a FORS2 measure (induced b* $\succ$ *) on the class of random vectors*

$$\Lambda_P = \{X \in \Lambda / p = \min(p_1,...,p_n) > 1 : \left\| p_X^{(1)}, (p_1,...p_n)^{(b_1,...b_n)} \right\| < +\infty \tag{3}$$

In addition $\rho_X^{(2)}$ *is associated with the following new FORS2 ordering induced by the previous on* $\succ$ *defined for every* $\alpha \geq \mathbf{1}$ *, and* $\forall X,Y \in \Lambda_{p,(\alpha)} = \left\{ Z \in \Lambda_p : \left\| \int_1^p ... \int_1^p \prod_{i=1}^{n} t_i^{\alpha_i - 1} d\rho_z^{(2)}, (t_1,..t_n) \right\| < \infty \right\}$ *we get:*

$$X \underset{\succ,\alpha}{FORS} 2Y \text{ if and only if } \rho_{X,\alpha}^{(2)}(u) \leq \rho_{Y,\alpha}^{(2)}(u) \cdot \forall u \in [1,p]^n \tag{4}$$

To proof this theorem we need to use the multivariate Mellin trasform that identify the common multivariate distribution when the second level of functionals are identical. Typical application of this theorem is represented by the moment orderings introduced in the Petronio et al. 2013.

3. Orderings among markets: an empirical comparison among the US, UK and German stock markets

Multivariate orderings can have several applications in economics and finance. In this section we discuss a possible application in ordering financial markets by the point of view of investors who has to choose the main market in which investing. With this aim we need to give some possible alternative definitions of orderings among financial markets/sectors.

Let us assume there are two markets: A with *n* assets, and B with *m* assets. Assume, the vector of the positions taken by an investor in the *n* risky assets of market A is denoted by $x = [x_1, \dots, x_n]'$ and similarly the vector of the positions taken by an investor in the *m* risky assets of market B is denoted by $y = [y_1, \dots, y_m]'$. We assume that no short sales are allowed.

Definition 3 *We say that a market/sector A with n assets strongly dominates another market/sector B with m assets with respect to a multivariate FORS ordering if for any vector of returns Y_B of $t \leq s = min(m, n)$ assets of market/sector B there exists a vector X_A of market/sector A such that X_A FORS Y_B . Similarly we say that a market/sector A with n assets weakly dominates another market/sector B with m assets with respect to the FORS ordering if for any given portfolio of gross returns $y'Y_B$ of market/sector B there exists a portfolio $x'X_A$ of the market/sector A such that $x'X_A$ FORS $y'Y_B$.*

Example 2. Suppose that the return distributions of markets A and B are jointly elliptically distributed. Suppose the markets have the same number of assets *n*, vectors of averages μ_A, and μ_B and dispersion matrixes Q_A and Q_B such that $\mu_A \geq \mu_B$ and $Q_A - Q_B$ is negative semidefinite. Then market A strongly dominates market B with respect to *the increasing concave multivariate order* (Muller and Stoyan 2002). Moreover market A weakly dominates market B with respect to the concave order since portfolio $x'\mu_A \geq x'\mu_B$ and $x'Q_A x \leq x' Q_B x$ for any vector $x \geq 0$. Observe that the weakly dominance between the markets is also know in ordering literature (Muller and Stoyan, 2002) as *the increasing positive linear concave multivariate order*.

Example 2 can be use in financial applications. In particular, if we assume that the returns of different markets are jointly elliptically distributed and they are uniquely determined by a risk measure and a reward measure, we can order the markets in a reward-risk framework. This observation is used in the following empirical analysis.

In order to identify the dominance among different markets we compare the reward-risk investor's choices of three different stock markets: US (Nyse, Nasdaq); UK (London stock exchange) and German (Frankfurt and Berlin). We consider all the returns in USD. Since it is not easy to prove the strong stochastic dominance among markets, then we try to evaluate the weakly stochastic dominance among the markets observing if one market dominates the other in a reward risk framework. Clearly we suppose that the distributional assumptions of Example 2 are verified for all the three markets. In particular, as reward measure we use the mean, while as risk measure we use either the variance or the Conditional Value-at-Risk, *CVaR*, or average value-at-risk,[2] expressed as:

$$CVaR_\alpha(X) = \frac{-1}{\alpha} \int_0^\alpha F_X^{-1}(u)du. \qquad (5)$$

In the following empirical analysis we use $\alpha = 5\%$. We consider the stocks of the three markets starting from January 2003 till May 2013. Every three months (60 daily observations) we estimate the reward-risk efficient frontiers of the three markets using:

[2] See Szegö (2004) and the references therein.

a) the first 150 most traded (in average) assets which were active during the last 12 years (3000 daily historical observations);
b) the first 350 most traded (in average) assets which were active during the last 4 years (1000 daily historical observations).

Therefore every three months we use a moving window either of 12 years or of 4 years. In this analysis we consider a dynamic dataset whose data are taken from DataStream. We identify the most traded assets of each market computing the mean of the daily average of traded value of each asset that is given by: *Daily average of traded value = Closing price*$\times$*Daily volume*.

Once that the mean of the daily average of traded value is computed over the historical period of observation (that is either 12 years or 4 years) we order them and we select the most traded in each market. Therefore, every 60 days, starting from the first January 2003, we fit the mean risk efficient frontiers of the three different markets for their oldest and the youngest firms. With this double comparison we evaluate the dynamicity of each market comparing the contributions of the recent firms and of the oldest ones.

Thus, at the k-th recalibration time (k = 1, 2,…,45), the following steps are performed:

Step 1 Preselect the most traded assets for each market and for each class of firms (old and young).

Step 2 Fit the mean risk efficient frontier solving the optimization problem for 30 levels of mean m:

$$\min_x \rho(x'z)$$
$$s.t.$$
$$x'e = 1; \quad x'E(z) = m$$
$$x_i \geq 0; i = 1,...,n$$

where $z = [z_1, , ..., z_n]'$ is the vector of the returns $\rho(x'z)$ is the risk measure (variance or CVaR) associated to the portfolio $x'z$.

The two steps are repeated for the three markets the two different class of firms and until the observations are available. The results of this empirical analysis are reported in Table 1 and Figures 1,2.

Table 1. Number of times (among 45) we observe that each single market reward-risk dominates another one.

	Analysis that uses the first 150 most traded assets active during the last 12 years					
	UG	UL	LU	GU	LG	GL
Mean-Variance	41	42	0	0	22	15
Mean-CVaR	14	0	0	15	21	15
	Analysis that uses the first 350 most traded assets active during the last 4 years					
	UG	UL	LU	GU	LG	GL
Mean-Variance	0	2	10	31	3	19
Mean-CVaR	0	2	10	17	2	11

Source: Authors' calculations using data from DataStream

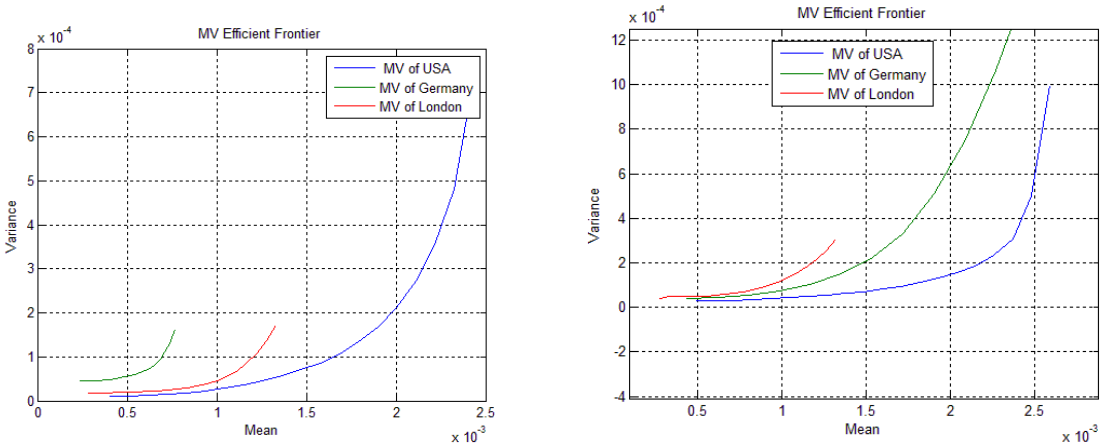
Table 1 reports the number of times a market dominates another one in terms of reward risk analysis during the decade January 2003- May 2013. We point out with:
1) UG the number of times the US market dominates the German one;
2) UL the number of times the US market dominates the London stock exchange market;
3) LU the number of times the London stock exchange market dominates the US market;
4) GU the number of times the German market dominates the US market;
5) LG the number of times the London stock exchange market dominates the German market;
6) GL the number of times the German market dominates the London stock exchange market.

First of all, we observe that there exists a strong difference between the comparison which uses the oldest firms with respect to the youngest of the markets. Considering the oldest firms we observe that generally US market dominates the other two in the mean variance framework but not always in the mean-CVaR framework. Moreover, we observe a different behavior before the crisis (2003- half 2008) and during the crisis (half 2008-2013). Before the crisis several times the oldest firms of the London stock exchange market present a much better behavior in terms of reward-risk than the analogous firms of the German market. While during the crisis it happen exactly the vice versa, where the German stock market presents sometimes better performance even to the US market.

This is also confirmed by Figures 1 and Figure 2 which reports the mean-risk efficient frontiers of some cases of observed dominance before and during the crisis, considering the firms existing during the last twelve years before the examination.

Figure 1: Mean-Variance dominance considering the firms existing during the last twelve years before the examination.

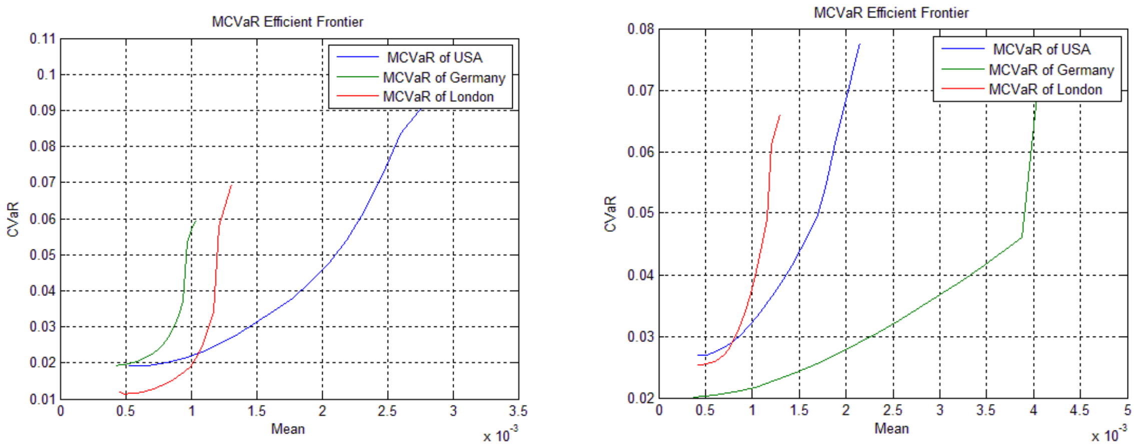
Example of Case 2003-2008 | Example of Case 2008-2013



Source: Authors' calculations using data from DataStream

Figure 2: Mean-CVaR dominance considering the firms existing during the last twelve years before the examination.

Example of Case 2003-2008 | Example of Case 2008-2013

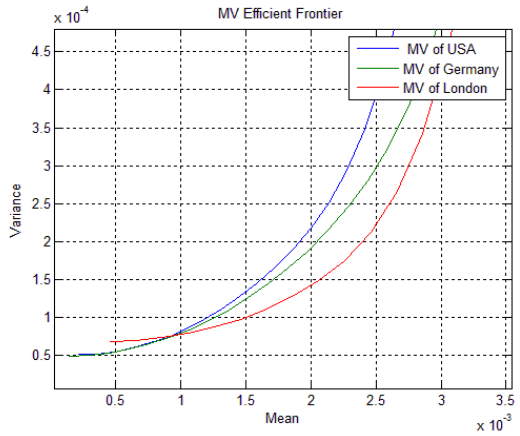


Source: Authors' calculations using data from DataStream

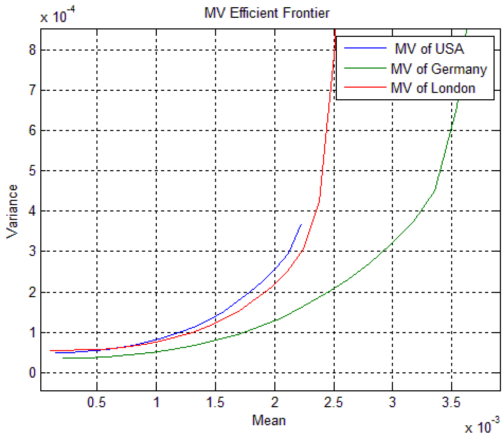
It is useful to observe that when we consider the first 350 most traded assets active during the last 4 years for each market the obtained results are completely different. Table 1 Figure 3 and Figure 4 show that the youngest firms of the German market present the best performance in particular during the crisis, while before the crisis (2003 – half 2008) the London stock exchange market sometimes dominates the US market and the German one. Moreover, using the youngest firms we observe that the dominance results in terms of mean variance or mean – CVaR are not too different.

Figure 3: Mean-Variance dominance considering the firms existing during the last four years before the examination.

Example of Case 2003-2008



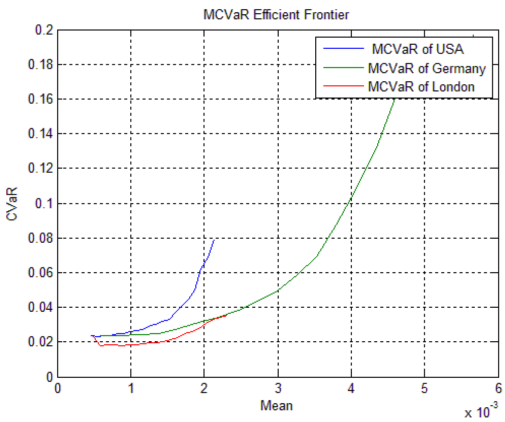
Example of Case 2008-2013



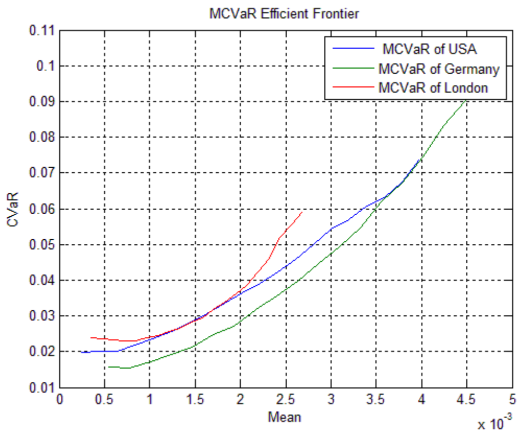
Source: Authors' calculations using data from DataStream

Figure 4: Mean-CVaR dominance considering the firms existing during the last four years before the examination.

Example of Case 2003-2008



Example of Case 2008-2013



Source: Authors' calculations using data from DataStream

Conclusion

FORS orderings can be used to extend several results of the theory of integral stochastic orderings that can be used to solve many financial problems. In this paper we propose an extension of the concept of multivariate FORS stochastic orderings and then we compare the reward risk behavior of three developed countries.

In this framework we propose a possible application where multivariate preferences are applied to order three financial stock markets (US, German and UK). In particular we identify the concept of dominance among different markets and we propose a first ex-post empirical comparison to evaluate their dominance relationships.

We observe that several times there exists reward risk dominance among the financial stock markets of different countries. Moreover, we also evaluate the dominance of the "oldest" and "youngest" firms of the different countries. Considering the US oldest firms generally dominates the ones of the other two countries in the mean variance framework but not always in the mean-CVaR framework. However, the youngest German firms present better performance in the analyzed decade (2003-2013). In particular, we observe a different behavior before the crisis (2003- half 2008) and during the crisis (half 2008-2013). Before the crisis several times the oldest and youngest firms of the

London stock exchange market present a much better behavior in terms of reward-risk than the analogous firms of the German market. While during the crisis exactly vice versa happens – the German stock market presents better performance even to the US market.

On the one hand, the methodology presented in this paper could be very useful for investors who want to optimize their international portfolio. In particular, this analysis can be generally applied to preselect the “best” markets where to invest. On the other hand, the strong differences observed between the two reward-risk approaches suggest that the optimal choices cannot be easily described by only two parameters. Thus, further analyses and comparisons that account of further distributional parameters seem to be necessary to better describe orderings among markets.

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