Yield Curve Modeling and its Applications for Post-Crisis Monetary Policy

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Abstract

Modern monetary policy requires new methods of extracting the market expectations through forward rated. To achieve their aims central banks utilize parametric forms that are used for fitting the term structure of interest rates.

The main purpose is to compare the yield curve models for three different fitting technique (objective functions) based on pricing errors, yield errors and prices divided by modified duration.

Since parametric models let built the yield curve only for a moment when data are taken to show its dynamic nature the daily changes of forward rates are to be analyzed. The paper focuses on the shapes of correlation surfaces build from the changes of implied forward rates which are received through parametric method of estimation. Additionally it shows the appropriate shapes of the form and surface and points the objective function for yield curve modeling in Polish market (based on inter-bank data). As a result, both the Nelson-Siegel and the Svensson model with approximation technique based on yield errors and prices divided by modified duration errors create forward rates which have the most well-behaved correlation matrix.

Keywords: yield curve estimation, parsimonious models, market expectations
JEL codes: C53, C92, E43, E58

1. Introduction

There are two different approaches how to understand the term structure of interest rates. First of them refers it as the relationship between the yield to maturity of default-free zero coupon securities and their maturities (Sundaresan, 2009). Second, presented by Nawalkha, Soto, Beliaeva (2004), suggests additionally a need of the same credit quality for a whole set of chosen assets. The graph, that plots the yields and fills in the gaps between the discrete values is called the yield curve (Choudhry, 2004). The shape therefore is investigated both by market participants and central bankers who are interested in extraction of expectations involved there. The problem is how to construct the yield curve from the term structure of interest rates to receive a precise picture of actual market situation. Additionally an ideal model should let extract market expectations easily from the implied forward rates.

In the beginning, monetary policy utilized the discount function as an estimating function (McCulloch, 1971), then splines were taken into account (Fisher, Nychka, Zervos, 1995) and finally parsimonious forms shown by Nelson-Siegel (1987) and Svensson (1994) became popular. Nowadays, central banks use ether cubic splines or parsimonious models, mainly because they easily fulfil the yield curve assumptions especially by ability to create four most popular shapes: positive, negative, flat and humped. Financial crisis and its consequences made to revise different approaches to yield curve modelling, and refresh their pros and cons.

The main aim of the paper is to describe a new assessment of yield curve adequacy based on correlation surface of implied forward rates (Rebbonato, 2002). The analysis covers two parsimonious models with three methods of parameter estimation. The research takes into account Polish inter-bank money market with data coming from period 2009-2012, during post-crisis period.

The paper is structured as follows: Section 2 provides a general overview of the implied forward rate extraction with Nelson-Siegel and Svensson model, Section 3 shows the applications of a correlation surface shape into yield curve's assessment. Section 4 is a final part which covers concluding remarks.

2. The implied forward rate modeling

In reality most of the term structures are not directly observable and they must be instead derived from asset prices. A common yield curve is typically built with a set of liquid assets; every instrument can be considered there as a portfolio of zero-coupon bonds (with the maturities adequate to the payment dates). Price of the zero-coupon bond is expressed as a discount factor which represents the relationship between the spot rate and the forward one. Since the asset's price, discount function, zero-coupon yield and implied forward rate are all algebraically related, knowing any one of these four values means that the other three can be readily computed:

$$P_{\tau}(\tau,t)=\delta(\tau,t)=e^{-i(\tau,t)\cdot(t-\tau)}=e^{-\int_{\tau}^{t}f_{\tau}(s)ds} \tag{1}$$

where: $P_{\tau}(\tau,t)$ - the price of an asset
$\delta(\tau,s)$ - the discount factor,
$f_{\tau}(s)$ - the implied forward rate,
τ - the moment when the forward transaction is calculated,
t - the expiring date.

Since the parsimonious models let receive the implied forward rate directly they play a crucial role in modern monetary policy. For the further analysis the Nelson-Siegel model with four parameters (NS_) and the Svensson model with six parameters (Sv_) were utilised. A set of parameters could be estimated by three different objective functions covering mean square errors minimising between:

- market prices and theoretical ones: $\frac{1}{k}\sum_{l=1}^{k}(P_l-\overline{P_l})^2\rightarrow\min$, (described as: NS_P, Sv_P);
- market and theoretical yields: $\frac{1}{k}\sum_{l=1}^{k}(i_l-\overline{i_l})^2\rightarrow\min$, (described as: NS_Y, Sv_Y);
- prices divided by duration $\sum_{l=1}^{k}\frac{(P_l-\overline{P_l})^2}{MD_l}\rightarrow\min$, (described as: NS_P/D, Sv_P/D);

where: $P_l-\overline{P_l}$ – a price error of *l*-th asset,
$i_l-\overline{i_l}$ – a yield error of *l*-th asset,
MD_l - modified duration of *l*-th asset,
k – number of assets

Taking into account three approximation methods, and assuming that the goodness of fit is not the most important measure, there is a need to apply new criteria which help to point out the most appropriate model. The question arises what kind of criteria should be chosen to achieve the suitable estimation.

Let τ be a number of the day for which the asset prices (data) for yield modelling are taken. Using the parsimonious models for the given day τ it is possible to calculate the instantaneous forward rate $f_{\tau}(s,s+\Delta s)$ with finite-length tenors $\Delta s>0$, the expiry at time s and maturing at time $s+\Delta s$. The assumed tenor is to be adequate to the length of open market operation (OMO) of the National Bank of Poland (NBP) for which it is the reference rate - a minimum yield of 7 days long NBP-bill.

Dealing with the general form for the implied forward rate $f_{\tau}(s,s+\Delta s)=\frac{1}{\Delta s}\cdot\ln\frac{\delta(\tau,s)}{\delta(\tau,s+\Delta s)}$ and assuming that the length is $\Delta s=\frac{7}{365}$ (yearly), a 7-days implied forward is given by:

$$f_\tau(s, s+\frac{7}{365}) = \frac{365}{7} \cdot \ln \frac{\delta(\tau, s)}{\delta(\tau, s+\frac{7}{365})} \tag{2}$$

where: $\delta(\tau, s)$ - discount factor,

$f_\tau(s, s+\frac{7}{365})$ - the implied 7-days forward rate,

τ - the moment when the forward transaction is calculated.

Since parametric models let built the yield curve only for a moment when data are taken to show its dynamic nature the daily changes of forward rates are to be analysed:

$$\Delta f_\tau(s, s+\tfrac{7}{365}) = \ln \frac{f_\tau(s, s+\frac{7}{365})}{f_{\tau-1}(s, s+\frac{7}{365})} \text{ for } n = 1,2,\ldots,17 . \tag{3}$$

The set represents the daily changes of 7-days long implied forward rate calculated at time τ with the expiring date in following days. It should be stressed that there are rates with the same length (tenor) but their expiring date is different. It could be written in the form of sequence:

$$\left\{\Delta f_\tau(s, s+\frac{7}{365})\right\}_{s=\frac{1}{365},\frac{2}{365},\ldots} . \tag{4}$$

2. The correlation surface

For the further analysis the example of seventeen implied forward rates were taken into account (in weekly intervals) based on WIBOR rate (Warsaw Inter Bank Offer Rate, the fixing rate from Polish inter-bank money market).

Following the calculation of the implied 7-days forward rate for seventeen different expiration dates $s = \frac{7}{365}, \frac{14}{365}, \ldots, \frac{119}{365}$, it can be shown as the vector:

$$\Delta \mathbf{f}_\tau(s, s+\tfrac{7}{365}) = \left[\Delta f_\tau(\tfrac{7}{365}; \tfrac{14}{365}); \Delta f_\tau(\tfrac{14}{365}; \tfrac{21}{365}); \ldots; \Delta f_\tau(\tfrac{119}{365}; \tfrac{126}{365})\right], \tag{5}$$

And when replacing $s \equiv \frac{7n}{365}$ for $n = 1,2,\ldots,17$:

$$\Delta \mathbf{f}_\tau(\tfrac{7n}{365}, \tfrac{7(n+1)}{365}) = \Delta \mathbf{f}_\tau(s, s+\tfrac{7}{365}) . \tag{6}$$

It is possible to calculate the correlation between two different rates:

$$\rho_{n,m} \equiv \rho_{n,m}\left(\Delta f_\tau(\frac{7n}{365}, \frac{7(n+1)}{365}); \Delta f_\tau(\frac{7m}{365}, \frac{7(m+1)}{365})\right) , \tag{7}$$

where: $\rho_{n,m}$ - correlation coefficient for $n, m = 1,2,\ldots,17$,

$\Delta f_\tau(\frac{7n}{365}, \frac{7(n+1)}{365})$ - the change of the forward rates following (3)

The calculated correlation coefficients $\rho_{n,m}$ create the correlation matrix $\mathbf{P}$ with seventeen rows and seventeen columns:

$$\mathbf{P} = \{\rho_{n,m}\}_{n=1,2,\ldots,17;\ m=1,2,\ldots,17} , \tag{8}$$

where: $\rho_{n,m} \equiv \rho_{n,m}\left(\Delta f_\tau(\frac{7n}{365}, \frac{7(n+1)}{365}); \Delta f_\tau(\frac{7m}{365}, \frac{7(m+1)}{365})\right)$, for $n, m = 1,2,\ldots,17$.

According to Rebonato (2002) and Wu (2009), the matrix $\mathbf{P}$ of coefficients $\rho_{n,m}$ should have following features:

[1] $\rho_{m,m} = 1$, for any m.

[2] $-1 \le \rho_{n,m} \le 1$, $n, m = 1,2,\ldots,17$.

[3] $\rho_{m,n} = \rho_{n,m}$.

[4] $\rho_{n,n+m}$ should be a decreasing function of m, for fixed n. This feature which is common in money market was firstly described by Brace, Gątarek, Musiela (1997) who named it as de-correlation effect and proved that it is analogous to the phenomenon observed among time series. It refers that the highest correlation is noticed between rates which dates of expiring are close (the distance $|n - m|$ is small). The bigger is the distance, the lower correlation is expected.

[5] $\lim_{m\to\infty} \rho_{n,n+m} = a > 0$, for fixed n.

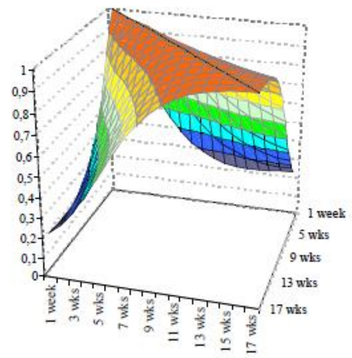
[6] $\rho_{n,n+m}$ should be an increasing function of n, for fixed m. This feature comes from the behavioural analysis market participants and stresses that the longer is the term to the expiring date the de-correlation effect is weaker. The market forecasts about the level of forward rates in far future are similar. In case of nearest future market participants have enough information to differ their expectations with the expiring term.

For the correlation surface, the following description is assumed:

n wks - the expiration of $f_\tau\left(\frac{7\cdot n}{365}, \frac{7(n+1)}{365}\right)$ where $n = 1,2,\ldots,17$.

The application of the correlation surface into modelling process let choose better form of approximation. If created surface fulfil [1]-[6] criteria it means that the functional form used for yield curie construction was appropriate. Any deviations suggest that one of the reasons could lies in the functional form chosen for approximation.

Figure 1: An ideal model of correlation surface

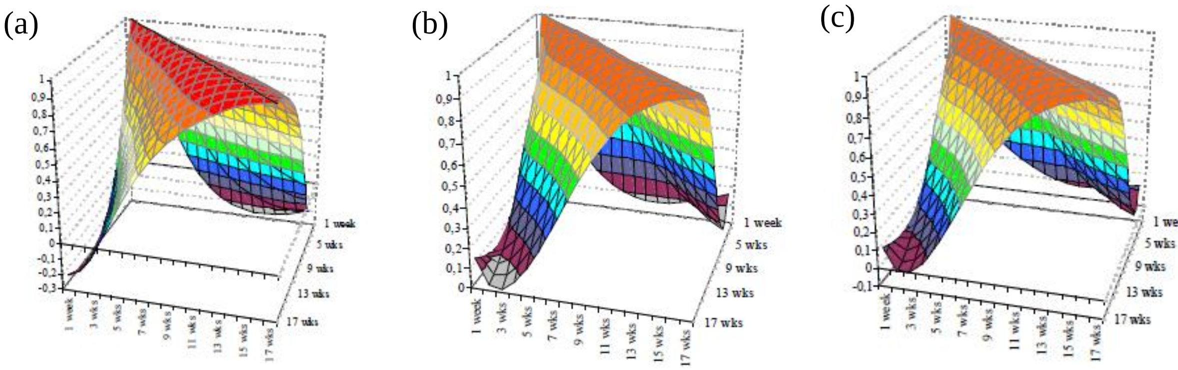


Source: author's calculations

For each day when data were available, six yield curves were constructed (for Nelson-Siegel and Svensson model with three approximation procedures). Therefore the implied 7-days forward with week intervals were calculated, next their daily changes, finally the correlation matrices and three-dimension surfaces from the elements of the matrix $\mathbf{P}$. The figures show that the implied 7-days forward rates have different volatility characteristics depending on the type of yield curve used.

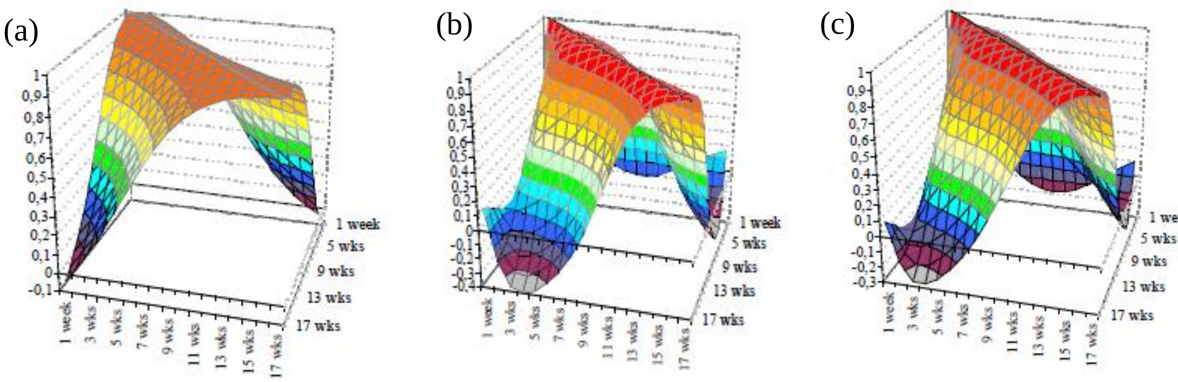
Two coordinates cover the expiring dates of different implied forward rates, when the third one the correlation coefficient value. The shapes let choose the best method of yield curve modelling. Both figure 2 and figure 3 were created basing on data from WIBOR market in 2009-2012.

Figure 2: The correlation surface for Nelson-Siegel model with an objective function based on prices (a), yields (b) and prices divided by duration (c).



Source: author’s calculations

Figure 3: The correlation surface for Svensson model with an objective function based on prices (a), yields (b) and prices divided by duration (c).



Source: author’s calculations

The figures show that the biggest difference between models is observable in longest maturity. The observation suggests that the models which should not be taken into account during yield curve modelling are these based on prices as an approximation method. They do not fulfil the criterion [5] and the limit of the correlation factor is going to negative value. The remaining models represent satisfying level of acceptance.

4. Conclusion

The post-crisis financial market let apply the correlation surface into yield curve analysis. It is an available tool for observations of the volatility level during time. Here it was presented the correlation surface made for a whole period. For the detailed analysis the period should be divided into shorter subsets and then it is possible to analyse the correlation surface during time.

The aim of this paper was to investigate the behaviour of forward rates obtained from fitting the WIBOR term structure. There was provided a detailed analysis of forward rates that were obtained using two parsimonious models with three different yield curve fitting techniques Based on approximately three years of daily data on Polish spot WIBOR rates, the correlation matrices were created and analysed with focusing on fulfilling chosen criteria. It has been shown that fitting techniques based on minimising the pricing errors should not have been used.

Not surprisingly therefore, both the Nelson-Siegel and the Svensson model with approximation technique based on yield errors and prices divided by modified duration errors create forward rates which have the most well-behaved correlation matrix.

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